Turbomachinery Research Laboratory
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Preliminary Aerodynamic Design of a Supercritical Carbon Dioxide Compressor Impeller for Waste Heat Recovery Applications

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Outline of the Presentation

- **Concept and Motivation**
  - Introduction of Supercritical Carbon dioxide Brayton cycles
  - Advantages and associated Challenges
- **Present Work Objectives**
- **Thermodynamic Design and Optimization of S-CO_2 Cycles**
  - Variants of the S-CO_2 cycle
  - Thermodynamic Cycle Modeling
  - Thermodynamic Cycle Optimization
- **Aerodynamic Design of S-CO_2 Centrifugal Compressors**
  - Sizing and Multi-Staging
  - A Brief Condensation Study
  - **Inverse Design Methodology**
  - Performance Analysis Methodology
- **Results and Inferences**

Introduction

- Depletion of Non-renewable energy sources + Rising energy requirements → Need of high-efficiency power conversion systems with small power block
- Supercritical Carbon Dioxide (S-CO_2) Brayton Cycles promises high thermal efficiencies (~50 %) at moderate temperature range (700˚C ~ 900˚C)
- Characteristics of S-CO_2 Cycles:
  (i) In-direct fired (Closed Brayton cycle)
  (ii) Operation close to the critical point
  (iii) Single-phase operation

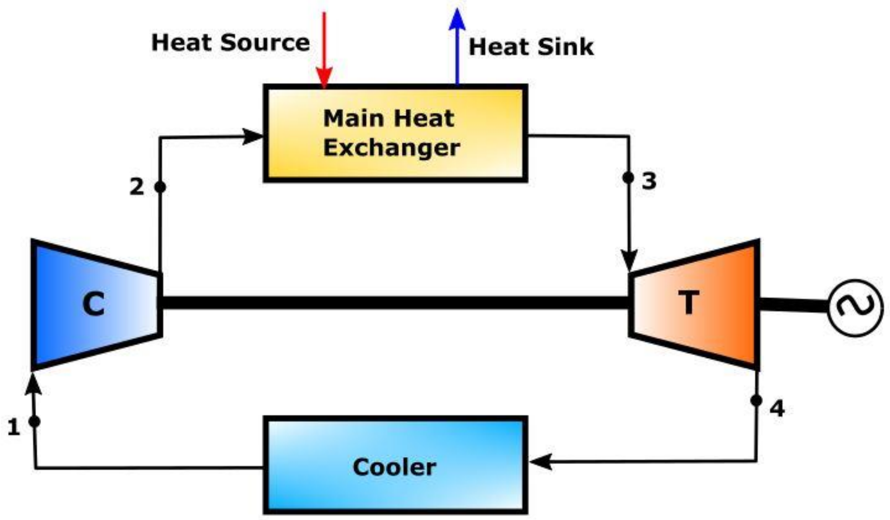


Figure 1: Cycle Layout of a simple S-CO_2 cycle

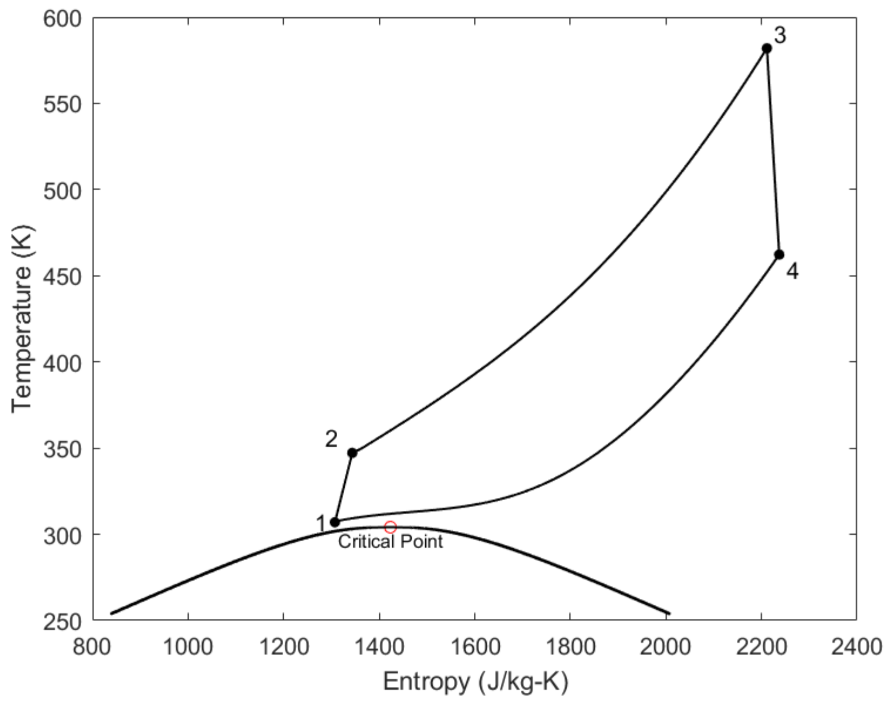


Figure 2: T-s Diagram of a simple S-CO_2 cycle

Advantages & Associated Challenges

Advantages

- Ideal Critical Point (P_c=73.77 bars , T_c=31.1˚C)
- Reduced Compression Work near the critical point → Higher Thermal Efficiency
- High fluid density → Smaller power block
- Wide Range of Applications: CSP, Nuclear, Fossil and Waste Heat Recovery

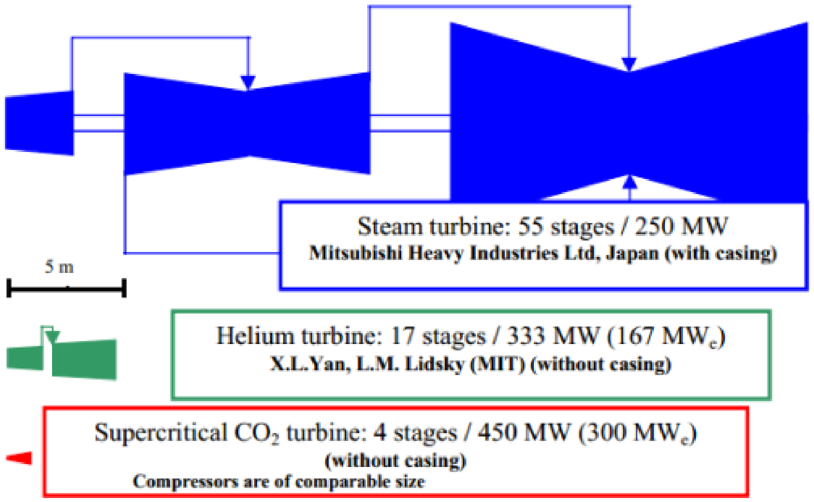


Figure 3: Comparison of Turbine Sizes | Source: Dostal (2004)

Challenges

- Commercial Viability
- Abrupt changes in thermodynamic properties near the critical point
- Impurity in working fluid → adverse effect on component performance

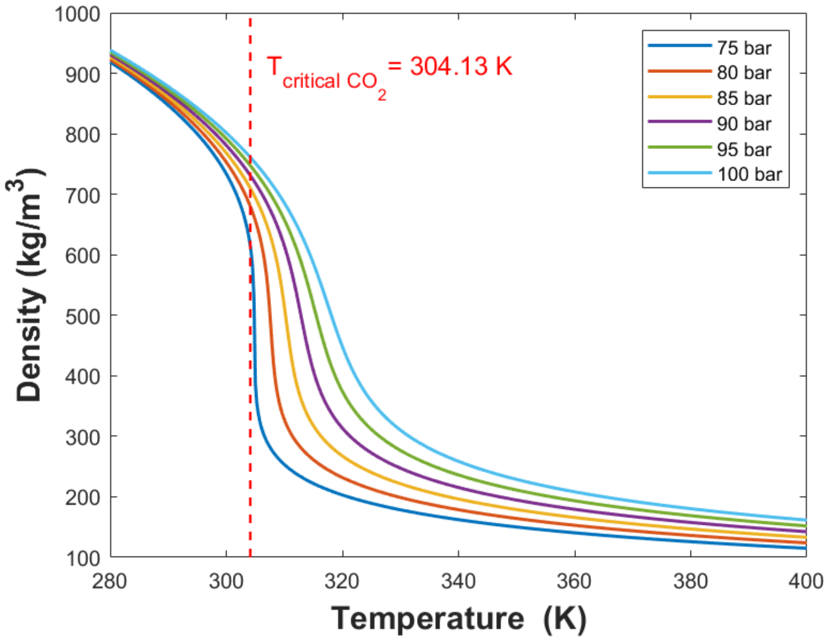


Figure 4: Density fluctuation near the critical point

Present Work Objectives

Motivation

- Waste Heat–to–Power technologies for low-to-medium heat source temperatures (300˚C - 600˚C) are limited
- Proposed S-CO_2 cycle → viable alternative to bottoming cycles such as steam-Rankine
- Most critical component for the cycle → Compressor

Objectives

- Design a S-CO_2 cycle to extract maximum power from a waste-heat source from a 10 - 15 MW source
- Perform a thermodynamic analysis and optimization of different configurations of the cycle
- Design the impeller of the compressor based on the optimized S-CO_2 cycle
- Develop a quick-yet-robust tool to analyze the performance of the S-CO_2 compressor

Table 1: Exhaust Gas Characteristic of the Waste-Heat Source

Working Fluid	**Air**
Inlet Temperature of Exhaust Gas	**450˚ C**
Inlet Pressure of Exhaust Gas	**1.02 atm**
Mass Flow Rate of Exhaust Gas	**50 kg/sec**
Ambient Temperature	**25˚ C**

Configurations of S-CO_2 Brayton Cycle

- Based on the literature, two prominent configurations of the S-CO_2 cycle are studied:

1. **Regenerative S-CO_2 Brayton Cycle (RC)**
2. **Re-compressive Regenerative S-CO_2 Brayton Cycle (RRC)**

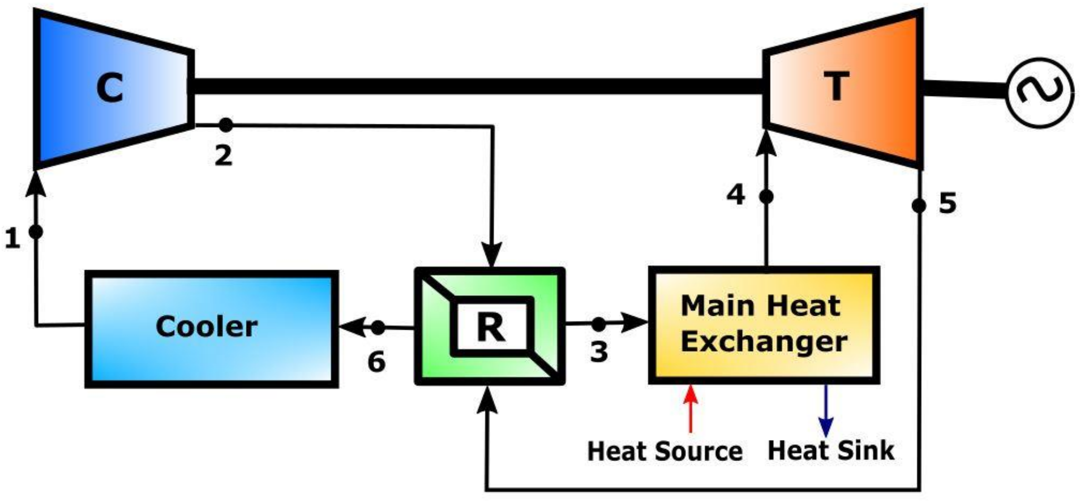


Figure 5: Cycle Layout of RC configuration

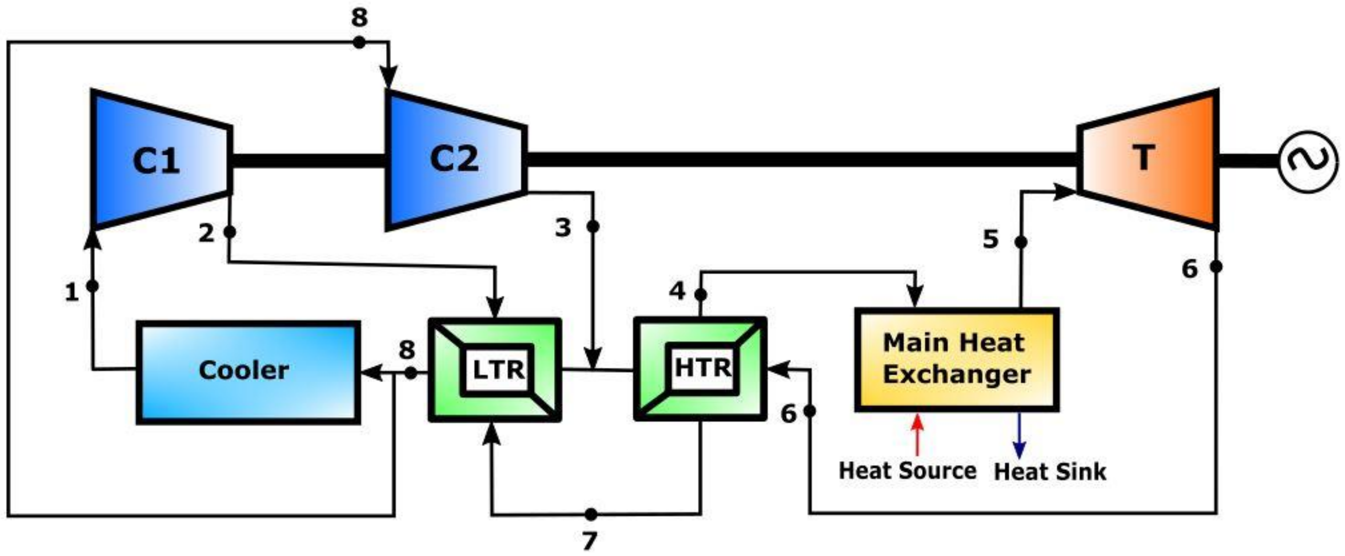


Figure 6: Cycle Layout of RRC configuration

Thermodynamic Cycle Modeling & Optimization

- **Modeling Assumptions**

1. Steady State Operation
2. Negligible change in K.E. and P.E.
3. Adiabatic Turbomachinery – Compressor and Turbine
4. No contaminants in Working Fluid

Methodology

Cycle Modeling

- **Input Variables: P_1 , P_2 , π_c , η_C , η_T**
- **Application of Mass and Energy Conservations**
- **Employing Modeling Assumptions**
- **Performance Parameters: η_{CYCLE} , $\dot{W}_{net}$**

Cycle Optimization

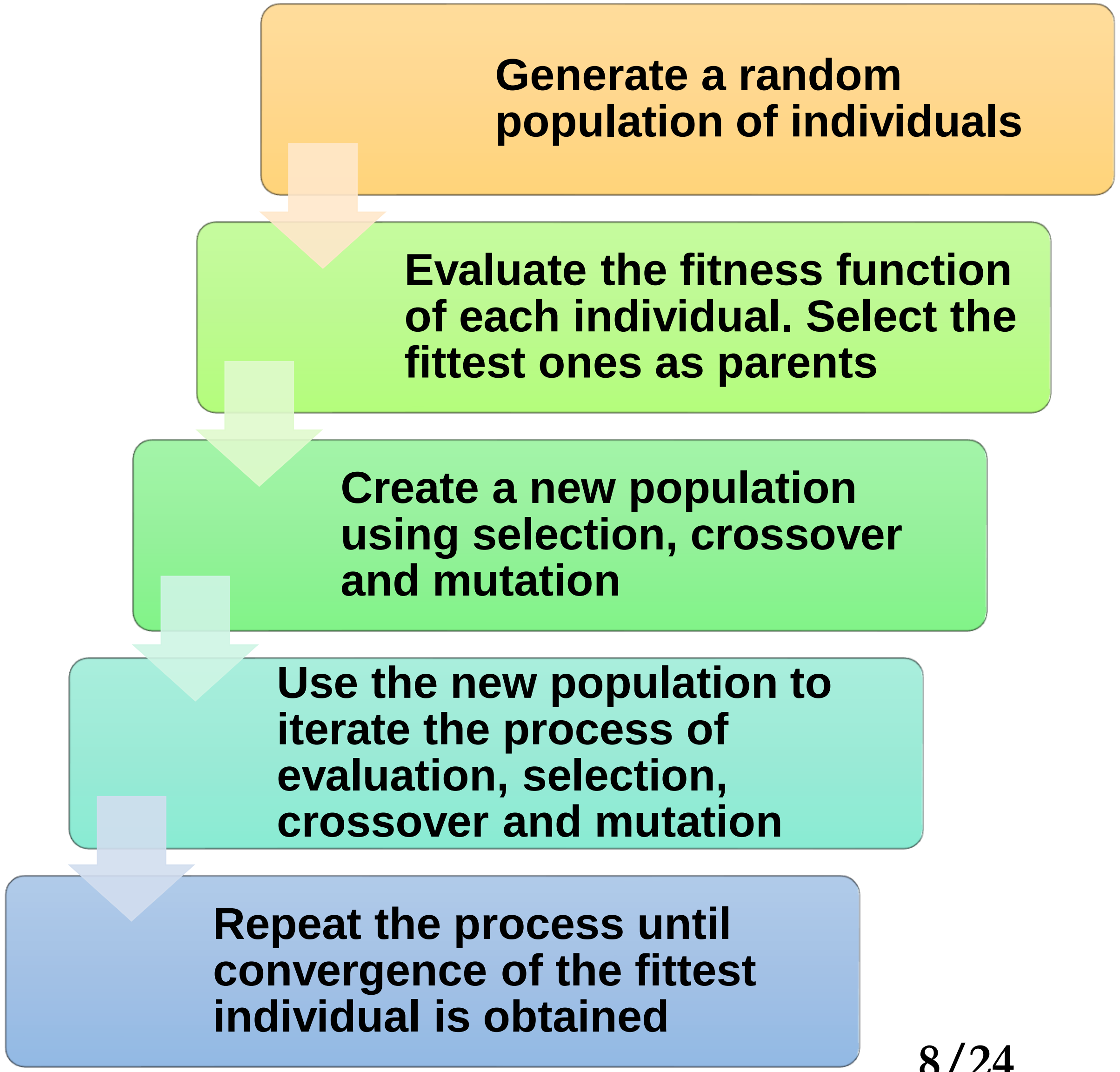
- **Genetic Algorithm (GA) methodology - to identify global optimum**
- **Input Design Variables: P_1 , T_1 , P_2 , $\dot{m}_{CO2}$, ΔT_{ttd}**
- **Defining Bounds for Design Variables**
- **Parameter to be optimized: $\dot{W}_{net}$**

Optimization Methodology: Genetic Algorithms

- Genetic Algorithm (GA) is a non-gradient based, stochastic optimization technique that is derived from natural selection
    - Based on bio-inspired operators such as Selection, Crossover and Mutation
    - Can deal with multiple decision variables; $y = f(x_1, x_2, x_3 \ldots x_n)$

- Operation of GA optimization can be summarized in five steps, as presented in the flow chart

- Variable to be optimized $\rightarrow \dot{W}_{net}$



Optimization Methodology: Input Parameters

- Certain inputs are provided prior to the initiation of the optimization process. These inputs includes the bounds of design variables, isentropic efficiencies and GA solver properties
- For the RRC configuration, an additional input parameter, the main compressor mass flow fraction, is added
- Bounds for the main compressor mass flow fraction ranges from 0 to 1
- The MATLAB GA optimization tool is employed along with RefProp to perform the optimization

Table 2: Input Parameters for the RC optimization

Input Design Parameters	Values
Compressor Efficiency, η_C	0.7
Turbine Efficiency, η_T	0.85
Fractional Pressure Drop, s	0.02
Waste Heat Stream Characteristics	Table 1

Table 3: Bounds of Design Variables for RC optimization

Design Variables	Lower Bound	Upper Bound
P_1 (bar)	74	100
P_2 (bar)	120	330
T_1 (K)	310	420
$\dot{m}_{CO_2}$ (kg/sec)	20	100
ΔT_{ttd} (K)	10	40

Optimization Results - I

Table 4: Optimized Design Variables of RC configuration

Optimized Design Variables	Values
Comp. Inlet Pressure, P_1 (bar)	88.44
Comp. Exit Pressure, P_2 (bar)	307.59
Comp. Inlet Temperature, T_1 (K)	310.13
Mass Flow Rate, $\dot{m}_{CO_2}$ (kg/sec)	48.76
Terminal Temp. Difference, ΔT_{ttd} (K)	10
$\dot{W}_{net}$ (MW)	3.13

Table 5: Optimized Design Variables of RRC configuration

Optimized Design Variables	Values
Comp. Inlet Pressure, P_1 (bar)	92.88
Comp. Exit Pressure, P_2 (bar)	303.00
Comp. Inlet Temperature, T_1 (K)	310
Mass Flow Rate, $\dot{m}_{CO_2}$ (kg/sec)	48.17
Terminal Temp. Difference, ΔT_{ttd} (K)	10.4
Main Comp. Mass Flow Fraction, f	1
$\dot{W}_{net}$ (MW)	2.94

- For a GA run with Generations = 100; Population size = 40
- For the RRC configuration, it is observed that the main compressor mass flow fraction approaches unity; thereby implying that the RRC configuration tends to RC configuration

Optimization Results - II

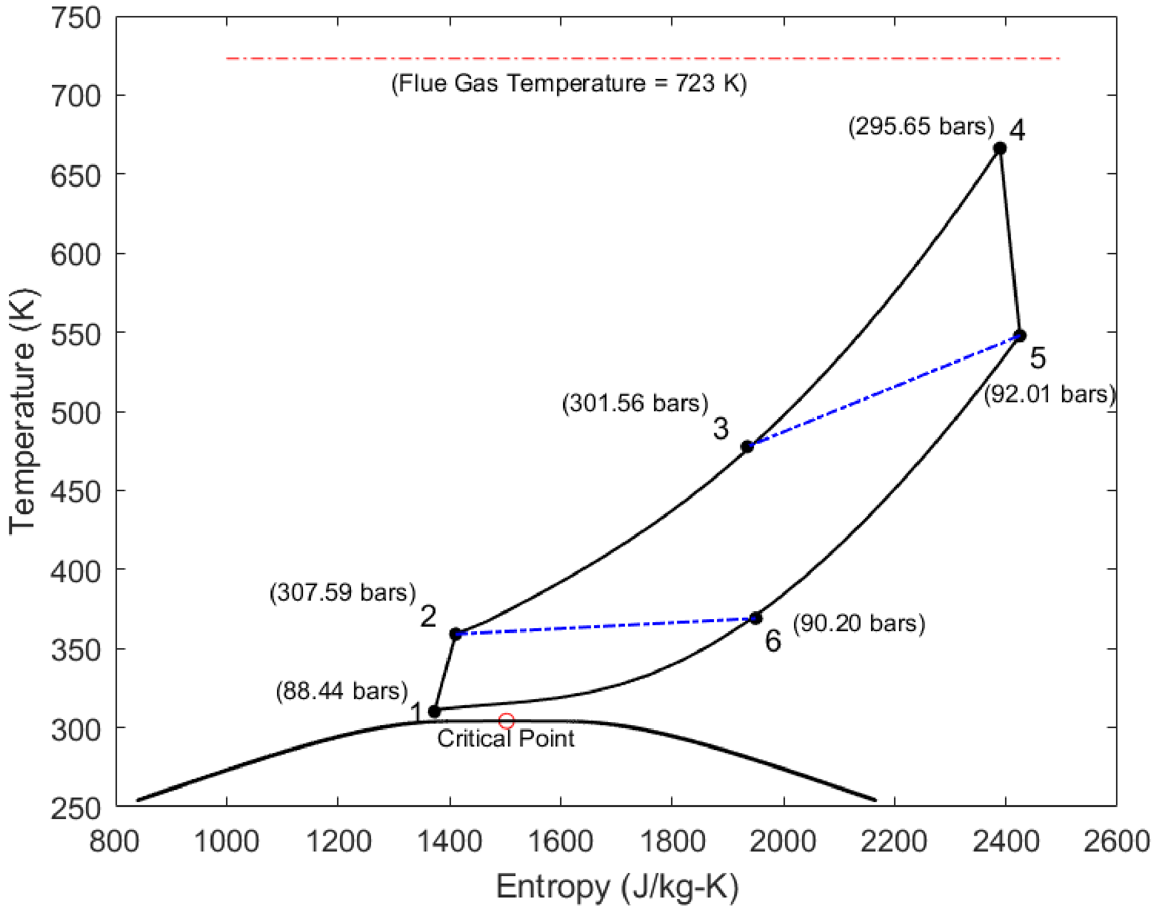


Figure 7: T-s Diagram of optimized RC configuration

- Turbine Inlet Temperature = 666.23 K
- Total Heat Recovered = 56%
- Thermal Efficiency of RC configuration = 25.09%
- Net Power Output = 3.13 MW

Aerodynamic Design of the S-CO_2 Compressor

- Using the thermodynamic states from optimized cycle, geometrical parameters of the compressor are calculated
- Performance model is coupled with the design methodology to dynamically modify the geometry and meet the desired performance

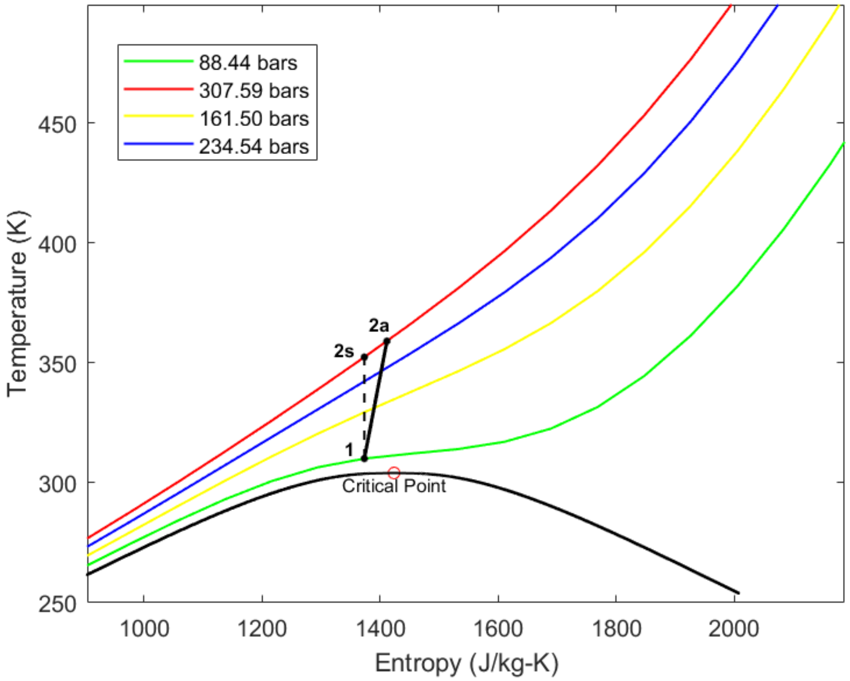


Figure 8: Compression Process of Optimized Cycle

S-CO_2 Compressor Design Process

1. **Compressor is sized using Balje's Diagram**
2. **Multi-staging of the compression process is performed**
3. **A brief condensation study is conducted for single and multi-stage compressors**
4. **Impeller inlet geometry is fixed using the criteria of no-condensation at throat**
5. **Impeller exit geometry is calculated using an Inverse design methodology**

Sizing and Multi-Staging

- Using the Balje's Diagram, appropriate specific speed and specific diameter are chosen to calculate the impeller RPM and diameter
- Assuming the compressor efficiency to be 0.7, and the optimum value of N_s = 0.7, Balje's diagram is used to obtain D_s and further the impeller diameter

$$\boldsymbol{N_s = \frac{\omega \times \dot{V}^{0.5}}{\Delta h_s{}^{0.75}}} \qquad \boldsymbol{D_s = \frac{d \times \Delta h_s{}^{0.25}}{\dot{V}^{\,0.5}}}$$

- Multi-Staging is performed by distributing the process such that each stage has equal specific work consumption and RPM

Table 6: Single Stage Compression System Parameters

Stage No.	Pressure Ratio	Isentropic Enthalpy Change (J/kg)	Volumetric Flow Rate (m^3/sec)	RPM	Impeller Diameter (m)
1	3.48	31814.66	0.083	47459.54	0.102

Table 7: Two-Stage Compression System Parameters

Stage No.	Pressure Ratio	Isentropic Enthalpy Change (J/kg)	Volumetric Flow Rate (m^3/sec)	RPM	Impeller Diameter (m)
1	2.17	15907.33	0.083	29219.61	0.122
2	1.59	15907.33	0.077	29219.61	0.151

A Brief Condensation Study

- Flow is likely to accelerate into two-phase dome in the throat of the compressor
- To quantify condensation, a Mach No. is defined: 'Acceleration Margin to Condensation' (AMC)
- For condensation to not occur, throat Mach No. should be less than AMC

$$M_{AMC} = \frac{\sqrt{2 \times (h_{o1} - h(s_{o1}, T_{sat}))}}{a(s_{o1}, T_{sat})}$$

- To calculate throat mach number, a value of inlet hub radius is assumed, while the inlet shroud radius is varied

$$d_{axis\,,\,min} = \sqrt[3]{\frac{16 \times \dot{W}_m}{\omega \times \pi \times \tau_m}}$$

- **For shaft material as AI-S-I-4330,** $d_{axis\,,min} = 3.44\ mm$
- **Considering a safety factor between 3-5, i.e., 4.5, chosen value of inlet hub radius,** $r_{hub-inlet} = 7.525\ mm$

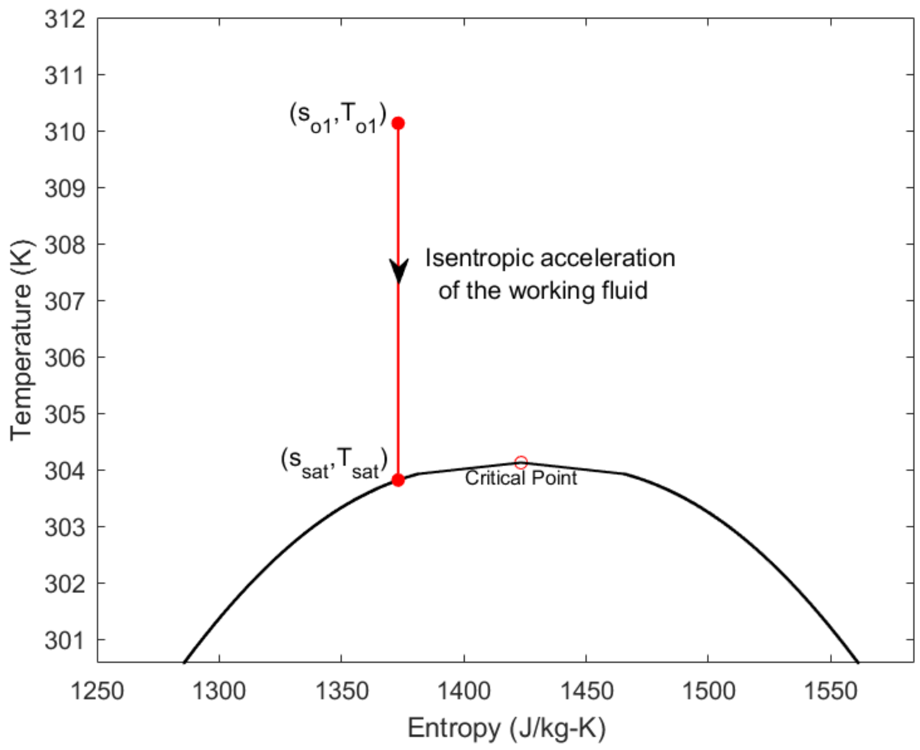


Figure 9: Fluid expansion to saturated state

A Brief Condensation Study - II

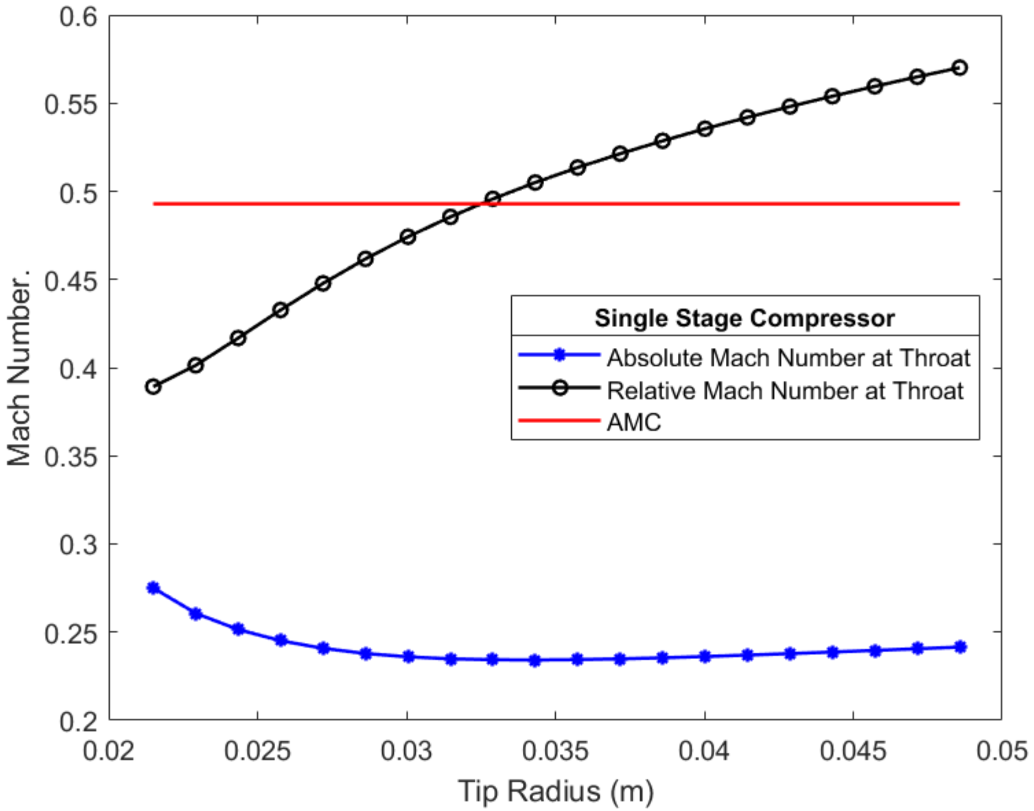


Figure 10: Variation of Mach No. with tip radius for single stage compressor

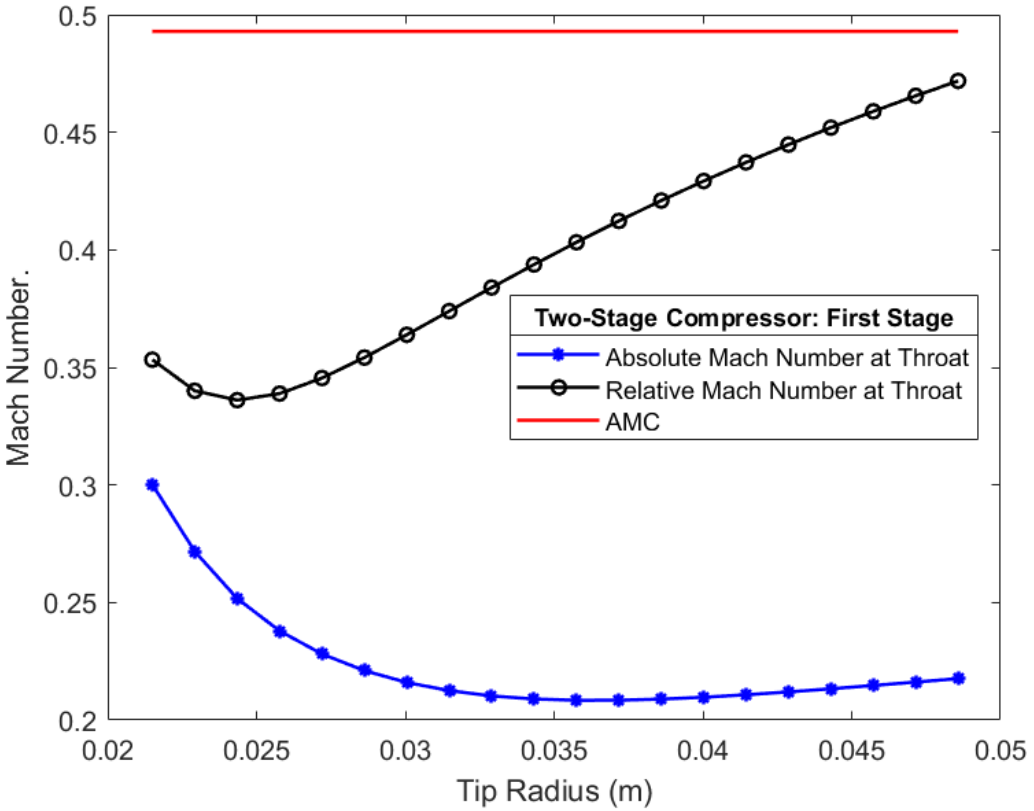


Figure 11: Variation of Mach No. with tip radius for two-stage compressor

- Based on the condensation analysis, a single stage compression process is selected with, $r_{shroud-inlet} = 21.5\ mm$ and $r_{hub-inlet} = 7.525\ mm$

Inverse Design Methodology

- Geometrical parameters of the compressor impeller are calculated using the thermodynamic state at inlet and exit of the compressor

- Design Methodology for Inducer and Impeller are modified from conventional design process for supercritical CO_2 fluid

Table 8: Input Parameters for Inverse Design Methodology

- **Inlet and Exit Stagnation Properties**
- **Mass Flow Rate**
- **RPM of the Impeller**
- **Impeller Tip Diameter (Balje's Diagram)**
- **Impeller Inlet Diameters (Condensation Study)**
- **Number of Blades**
- **Clearance Gap and Blade Thickness**

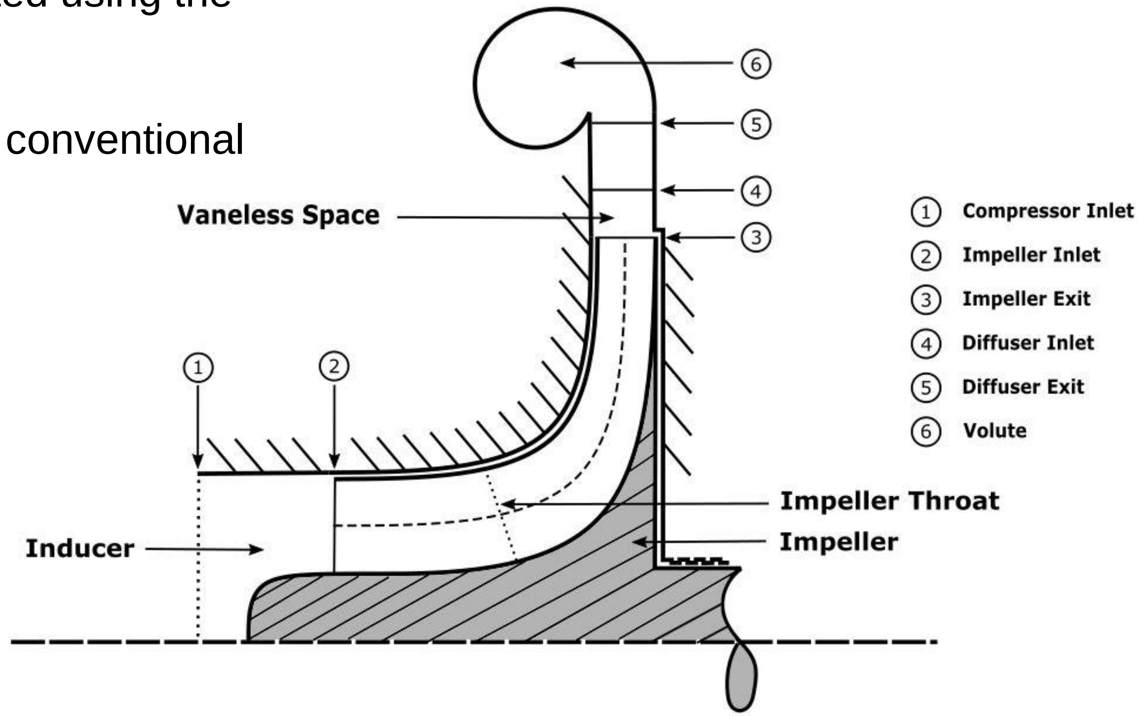


Figure 12: Highlighted Stations of a Centrifugal Compressor

$$\dot{m} = \rho \times (\vec{C} \cdot \vec{A}) \times (1 - B)$$

Mass Conservation Equation

$$\dot{Q} + \dot{W} = \dot{m} \times \Delta \left(e + \frac{P}{\rho} + \frac{C^2}{2}\right)$$

Energy Conservation Equation

$$h_{o2} - h_{o1} = \vec{C_2} \cdot \vec{U_2} - \vec{C_1} \cdot \vec{U_1}$$

Euler Turbine Equation

Inverse Design Methodology - II

- The primary target of the inducer design is to evaluate the velocity triangle at the inducer inlet
- The algorithm is initialized by assuming no losses
- Post evaluation of the velocity triangle, the blade metal angle is set equal to the relative velocity angle with the meridional direction, thereby assuming null incidence
- At the throat, it is ensured that the Mach number remains lower than the AMC, to avoid any condensation
- Post the calculation of geometry of the inducer, a performance analysis tool, adopted from Aungier's work, is employed to calculate the static pressure at the inducer exit
- The performance analysis tool takes the inlet thermodynamic conditions and geometry as input, and outputs the exit thermodynamic conditions

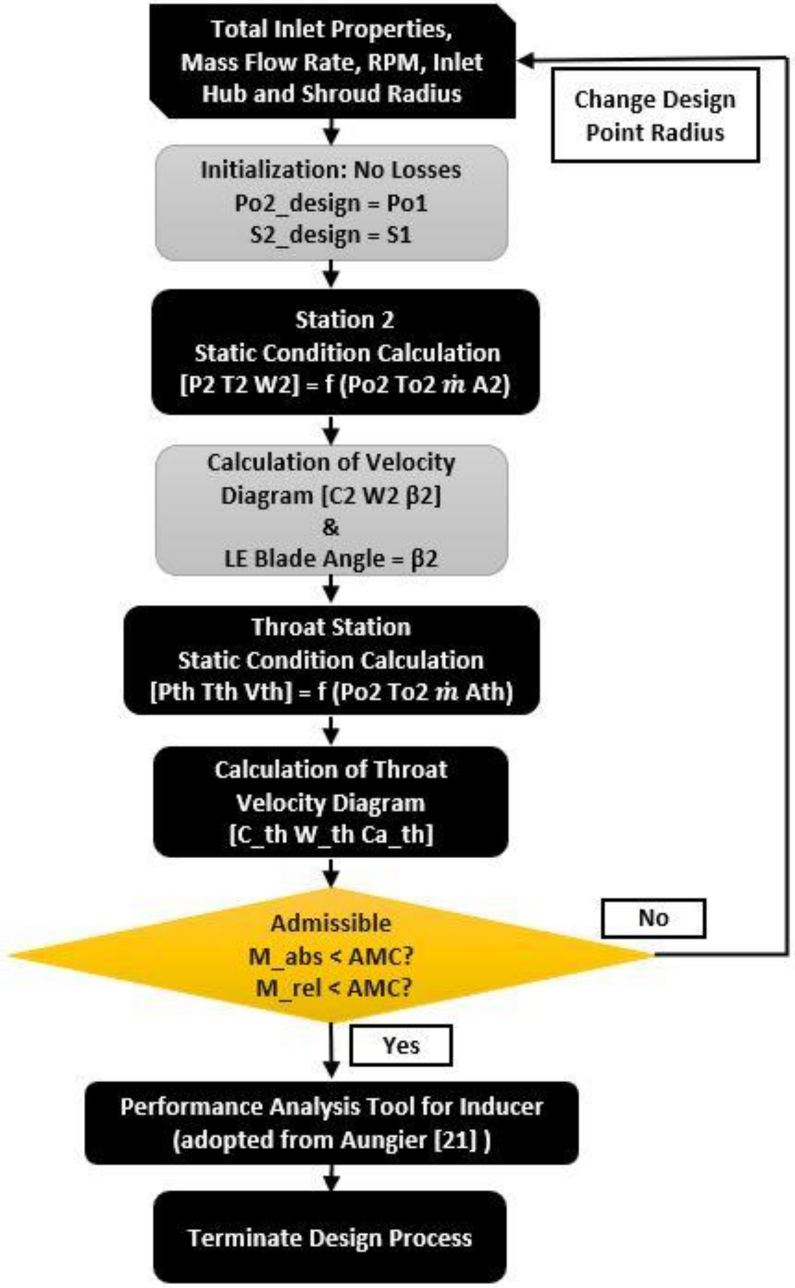


Figure 13: Algorithm for Inducer Design

Inverse Design Methodology - III

- Losses in the diffuser and volute are assumed to be of a constant (1%) value during the design process

$$\kappa_{diff-vol} = \frac{P_{o3} - P_{o6}}{P_{o3}}$$

- Performance parameters such as the slip factor is adopted from Weisner's correlation

$$\sigma_{weisner} = 1 - \frac{\sqrt{\sin(\beta_3)}}{z^{0.7}}$$

- A performance analysis tool for impeller, adopted from Aungier's work is employed in the algorithm

Table 9: Calculated Geometrical Parameters of the S-CO_2 Compressor

Number of Full Blades	**12**
Number of Splitter Blades	**0**
Blade Thickness at Impeller Leading Edge	**1 mm**
Blade Thickness at Impeller Trailing Edge	**1 mm**
Clearance Gap between the Shroud and Blade	**0.25 mm**

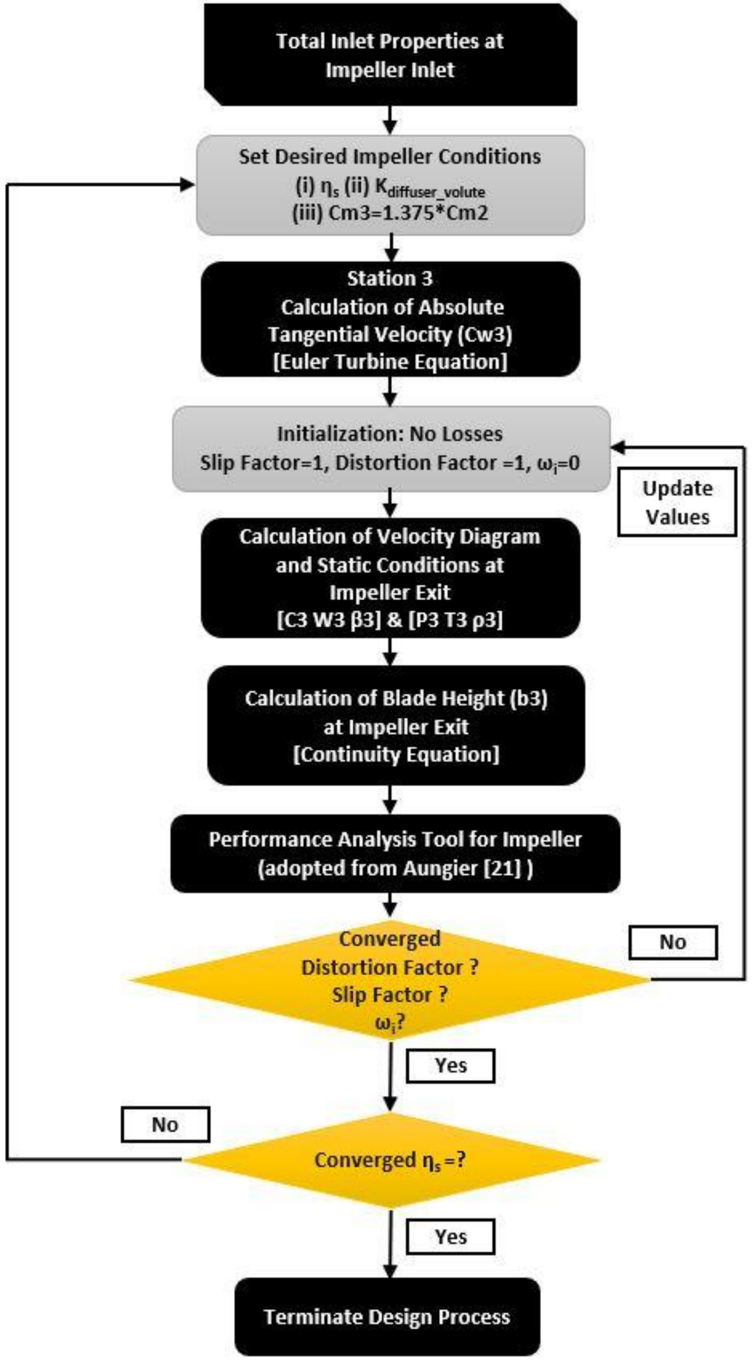


Figure 14: Algorithm for Impeller Design

Performance Analysis Methodology

Assumptions:

- Steady state
- Adiabatic
- Kinetic and potential changes negligible
- Quasi one-dimensional model
- No large fluctuations in properties span-wise

Table 10: Input and Output Parameters for Performance Model

Input Parameters	Output Parameters
• **Working Fluid**	• **Pressure Ratio**
• **Impeller Geometry**	• **Thermal Efficiency**
• **Inlet Stagnation Properties**	• **Exit Thermodynamic Conditions**
• **Mass Flow Rate**	• **Slip Factor, Blockage**
• **Shaft Speed**	• **Pressure Losses**

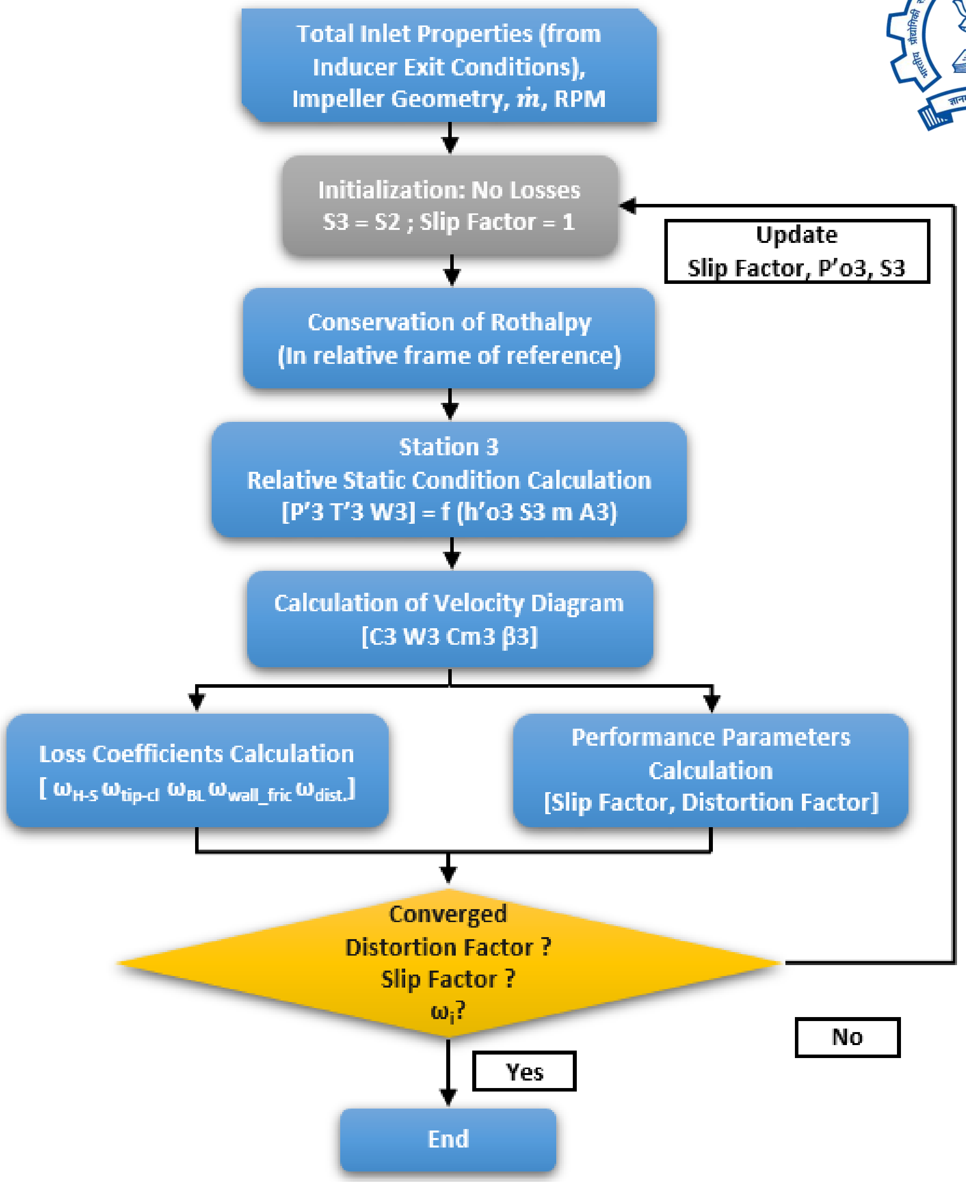


Figure 15: Algorithm for Impeller Performance Analysis

Validation: Performance Analysis Code

Table 11: Comparison of Model Performance with SNL Compressor Experimental Data

ω (RPM)	T_{o1} (K)	P_{o1} (bar)	$\dot{m}$ (kg/sec)	P_3 (exp.) (bar)	P_3 (model) (bar)	Error (%)
10000	305.5	76.76	0.454	76.76	77.614	1.113
20000	305.5	76.76	0.771	78.54	81.98	4.379
49000	306.3	78.54	1.816	94.25	98.13	4.116
60000	306.9	79.97	2.225	102.11	109.21	6.953
64900	307.9	82.11	2.406	108.53	116.17	7.039

- Mean deviation of 4.67% with a peak of 7.04% is observed, given the complexity of model
- Model is used to analyze performance of different impeller geometries
- Choking mass flow rates are not captured here because of numerical instability of the code (static properties fall into the two-phase dome)

Inverse Design Methodology: Results - I

Table 12: Calculated Geometrical Parameters of the S-CO_2 Compressor

Impeller Exit Tip Radius	50 mm
Blade Angle of the Impeller Leading Edge	46.44°
Blade Angle of the Impeller Trailing Edge	-11.74°
Axial Length of the Impeller	38.7mm
Full Length of the Impeller Blade	68.0mm
Blade Height at Impeller Inlet	13.725 mm
Blade Height at Impeller Exit	2.6 mm

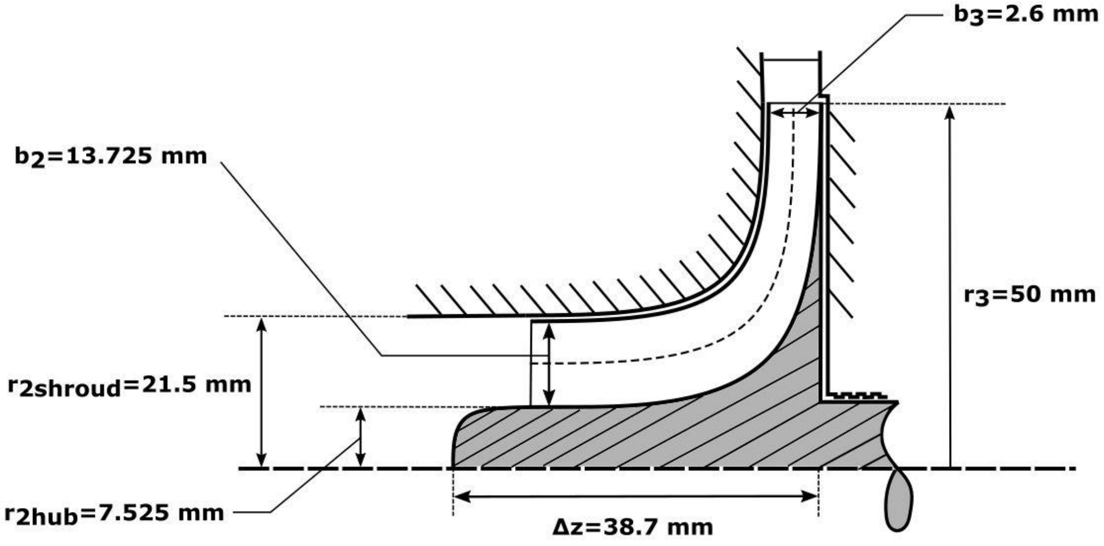


Figure 16: Resulting Geometry of the S-CO_2 Impeller

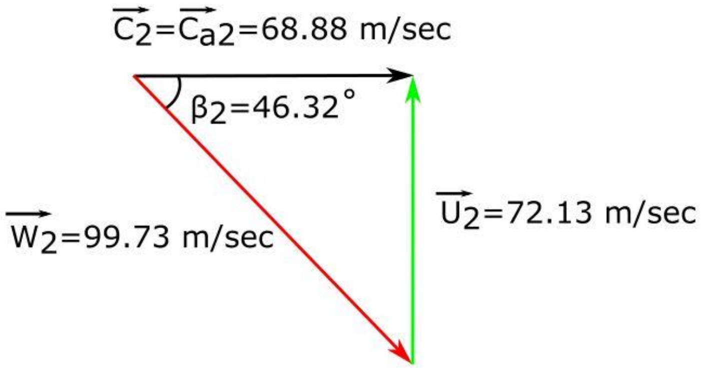


Figure 17: Velocity triangle at Impeller Inlet

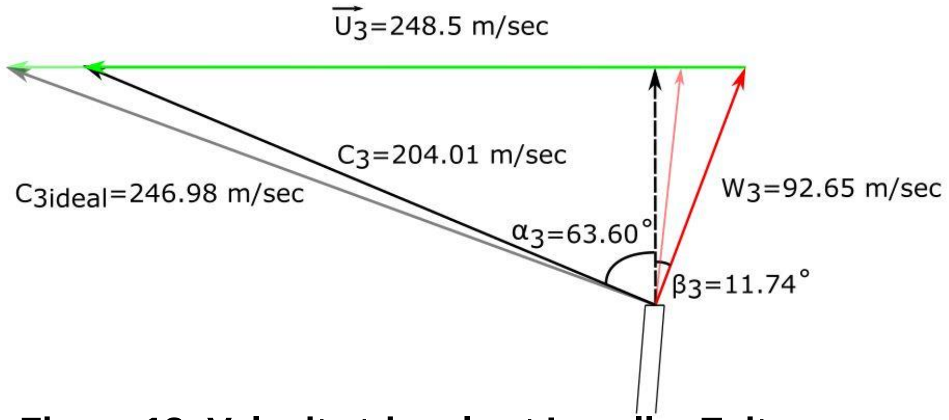


Figure 18: Velocity triangle at Impeller Exit

Inverse Design Methodology: Results - II

Table 13: Comparison of calculated thermodynamic properties with desired values

Thermodynamic Properties at Impeller Exit (at mean radius)	Desired Value (From Cycle Optimization)	Calculated Value (From Inverse Design Code)	Deviation (%)
P_o (bar)	307.59	319.44	3.85
T_o (K)	359.16	359.93	0.22
h_o (kJ/kg)	361.86	361.81	0.01
ρ_o (kg/m^3)	736.67	728.06	1.10
S (kJ/kg-K)	1.412	1.41	0.14

- Compressor Efficiency = 70.07%
- Slip Factor = 0.826
- Distortion Factor = 1.389

Conclusions

- It is found that for the pre-defined heat source, S-CO_2 RC configuration outperforms the RRC configuration. The RRC configuration tends to approach the RC configuration for low heat inputs.

- A brief condensation analysis ensures that the throat section of the compressor which is more prone to formation of liquid remains free of any condensation.

- S-CO_2 impeller delivers a slightly higher total pressure than desired. Total pressure loss in the diffuser and volute is not calculated and therefore, resulting value for the current design with a deviation of 3.85% should be admissible

- For the current design, Slip factor and Distortion factor with the value of 0.83 and 1.39 are admissible values. Also, the reported efficiency at the end of impeller design is 70.1% which meets the target compressor isentropic efficiency

Thank - You