

# Turbulence in non-ideal fluids

Rene Pecnik

Process and Energy Department,  
Delft University of Technology,  
The Netherlands



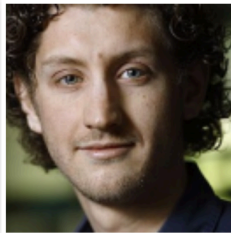
# Acknowledgments



**Dr. Ashish Patel**  
Graduated June 2017  
Currently in California, US



**Dr. Hassan Nemati**  
Graduated in Nov. 2016



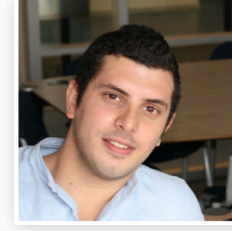
**Dr. Jurriaan Peeters**  
Graduated Nov. 2016  
Assistant Professor  
TU Delft, NL



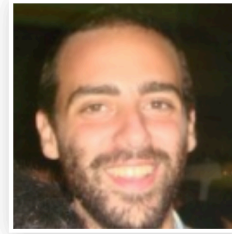
**Dr. Jie Ren**  
now in Nottingham, UK



**Gustavo Otero Rodriguez**  
PhD student  
☎ +31 15 27 83884  
✉ G.J.OteroRodriguez@...



**Simone Silvestri**  
PhD student  
☎ +31 15 27 86628  
✉ S.Silvestri@tudelft.nl



**Dr. Enrico Rinaldi**  
Graduated Sept. 2015  
Currently postdoc at KTH,  
Linne FLOW Centre,  
Stockholm



**Dr. Uttiya Sengupta**  
Graduated in Feb. 2019



# Motivation

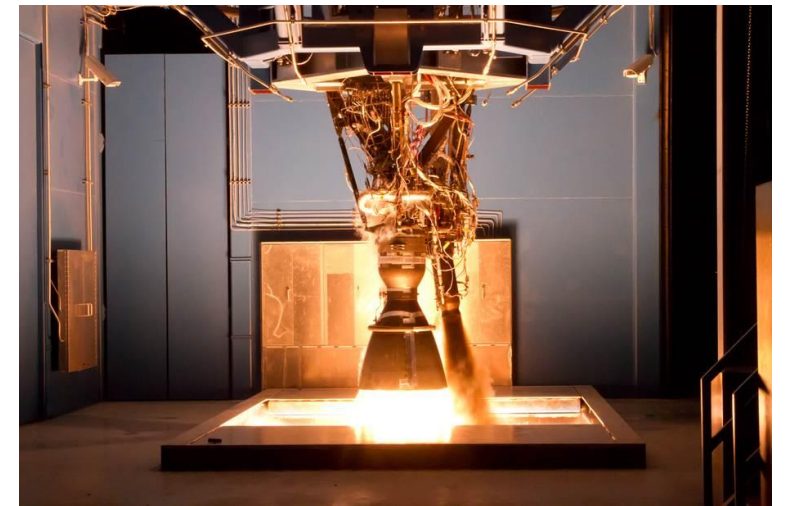
The continuous demand to increase

efficiency in **energy conversion** systems

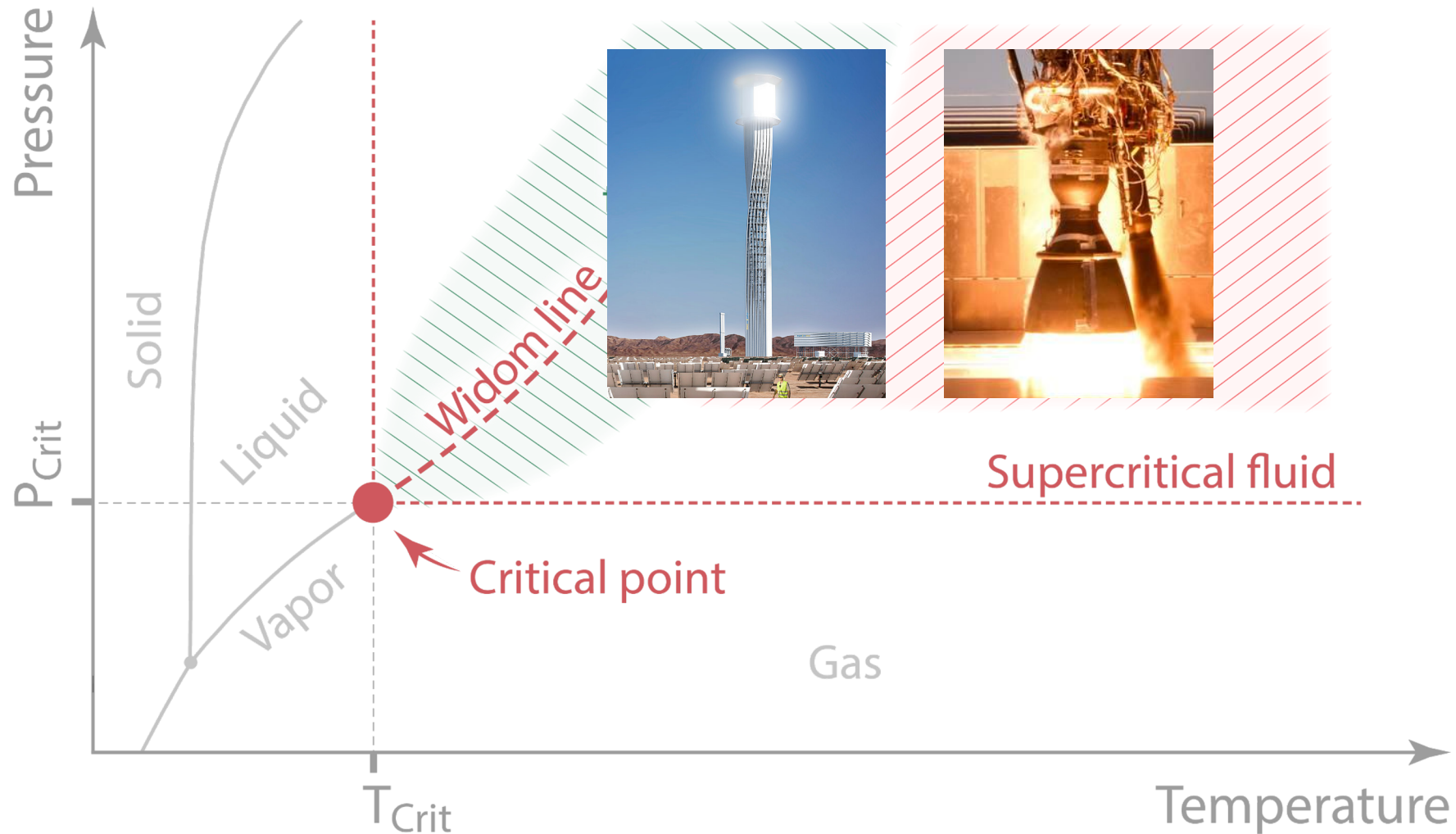
&

productivity in **chemical processes**

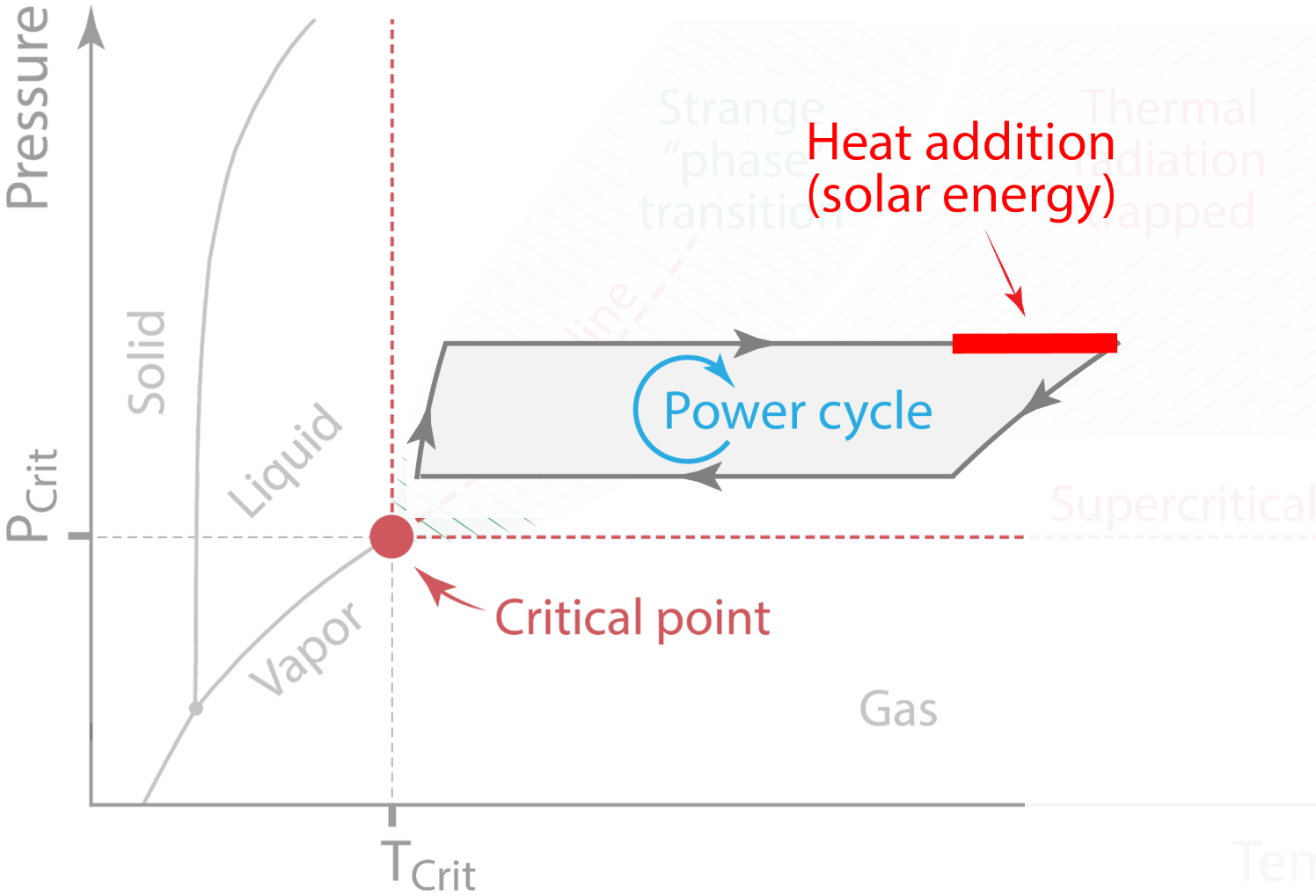
forces engineers to use fluids at increasingly  
higher **pressures** and **temperatures!**



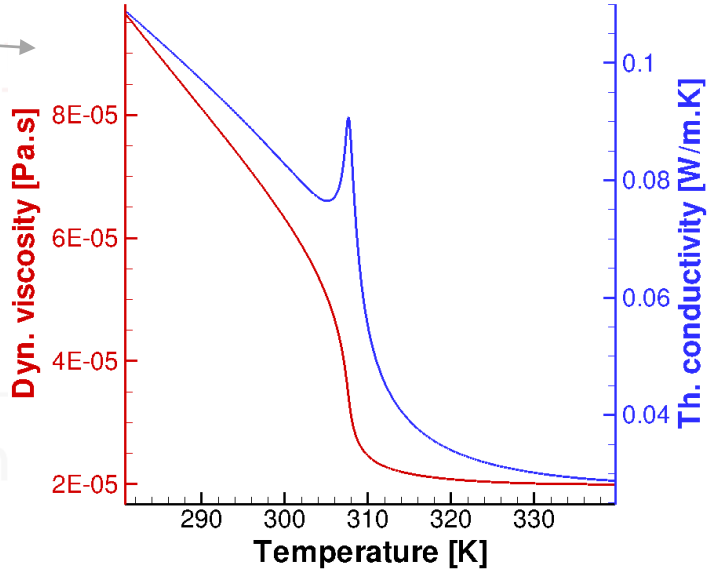
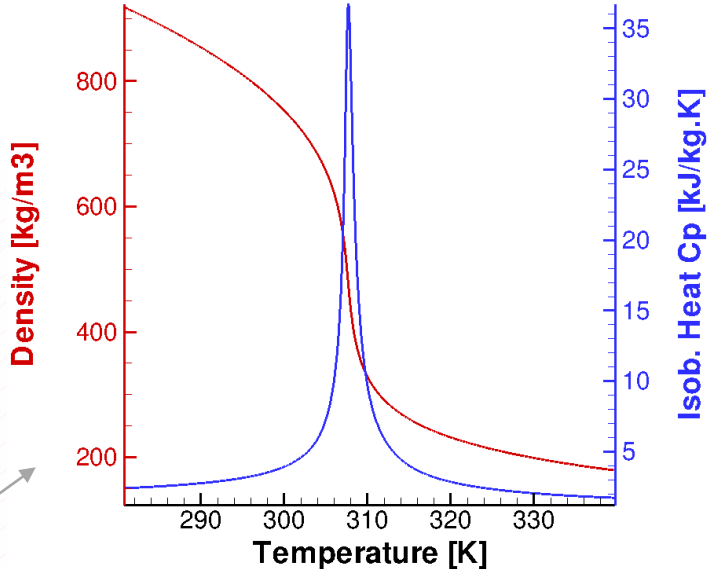
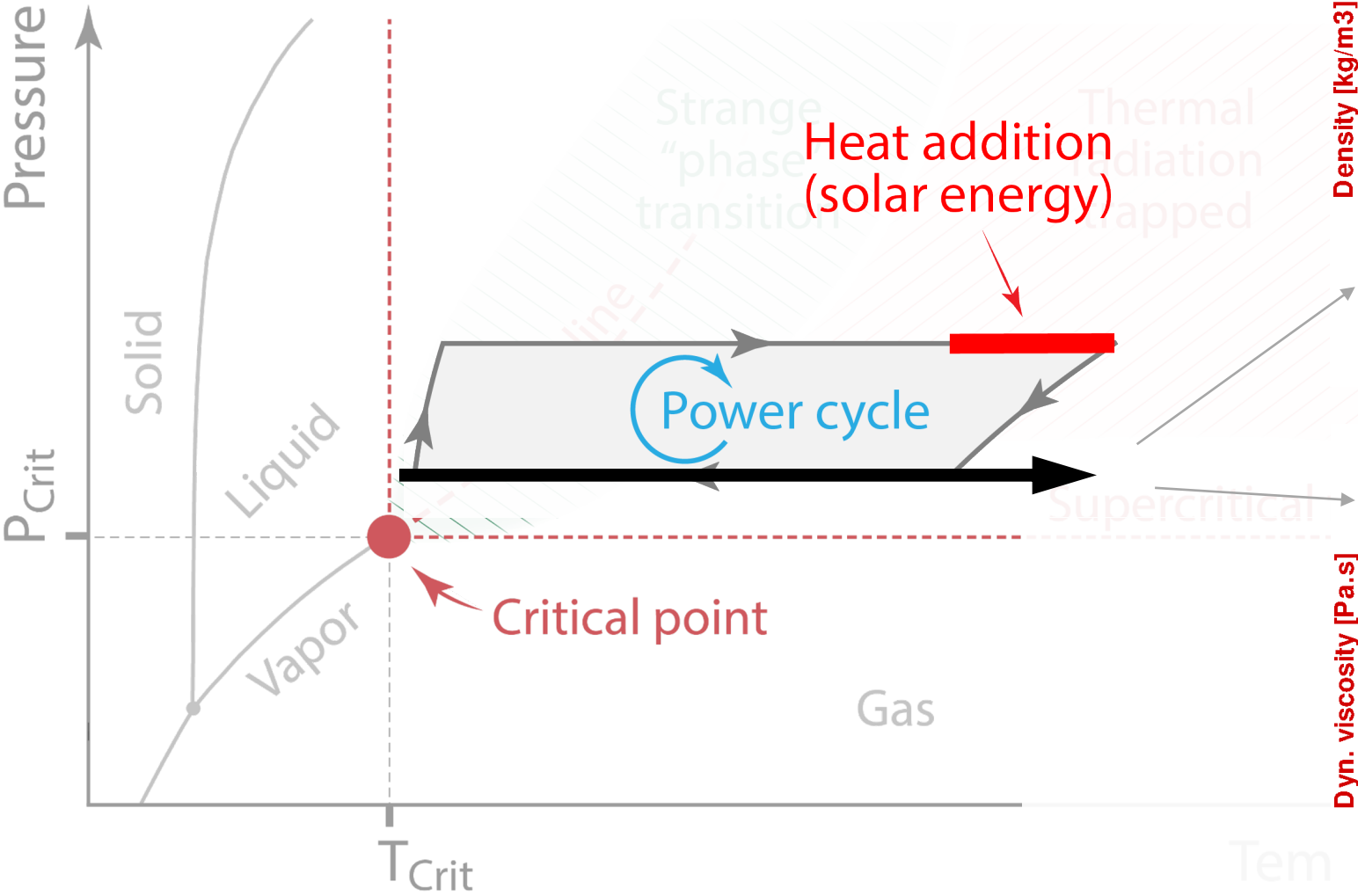
# Phase diagram of an arbitrary substance



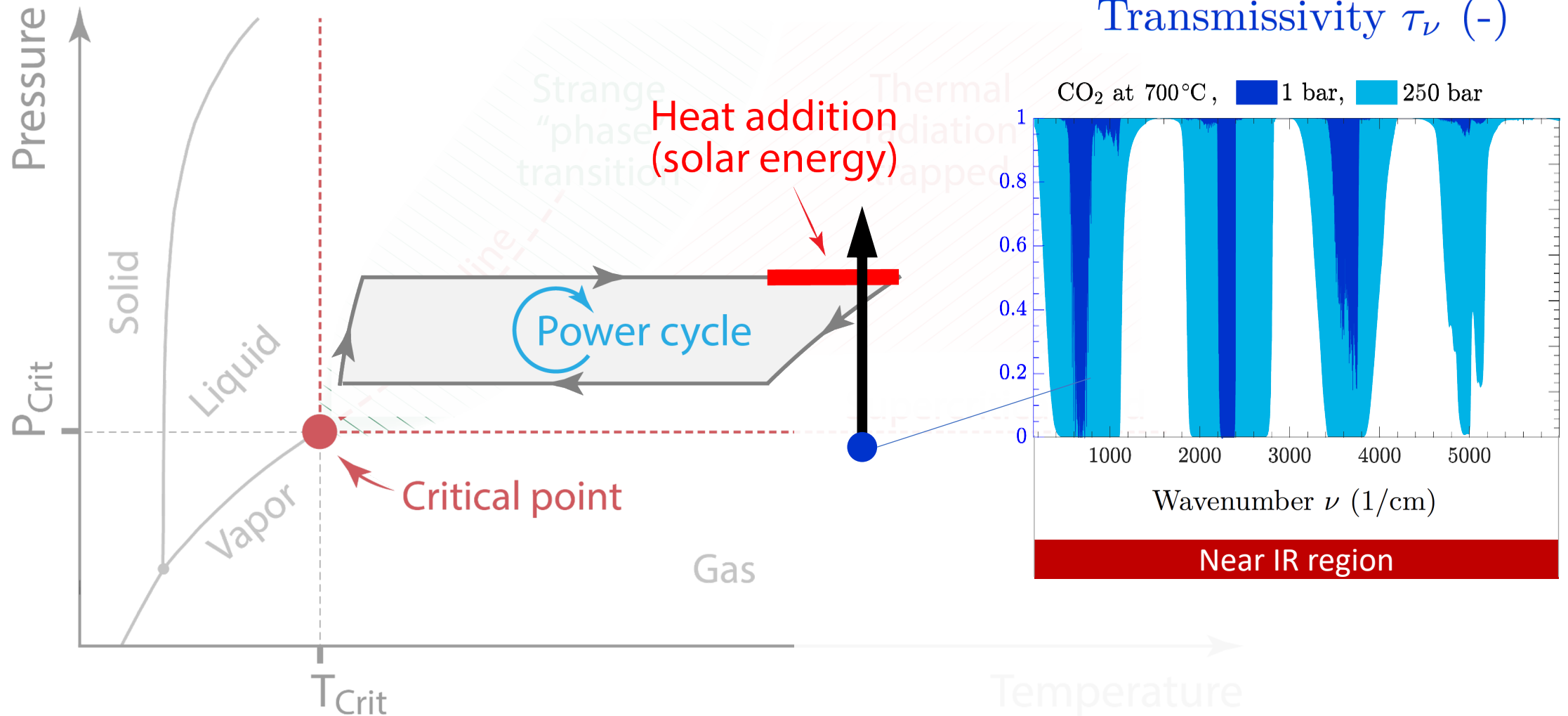
# Supercritical power cycle



# Properties of CO2 @ 80 bar

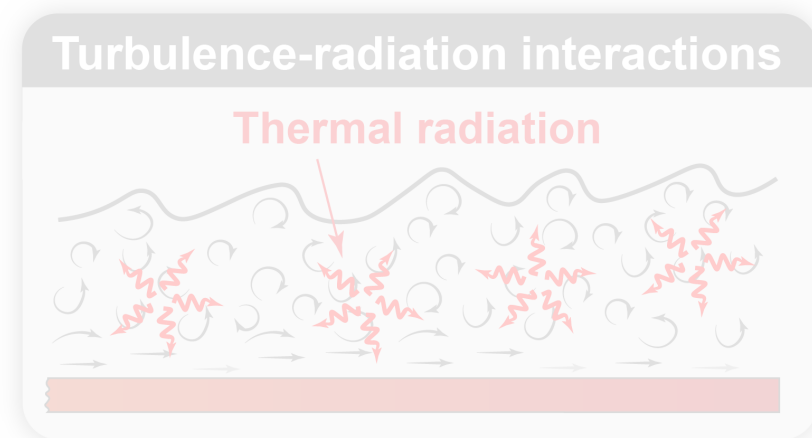
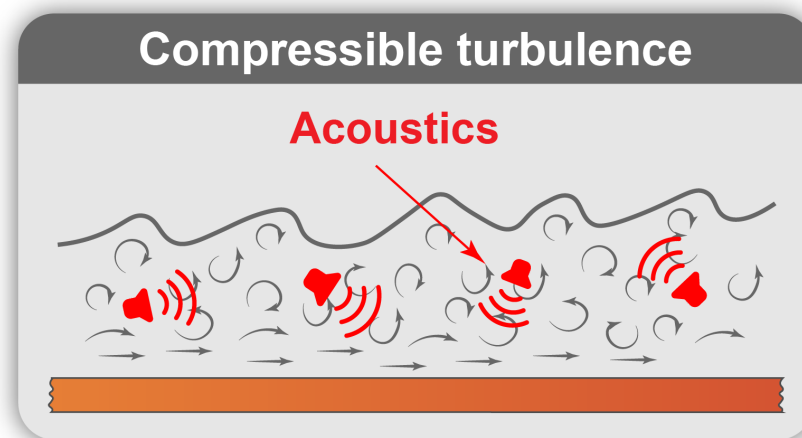
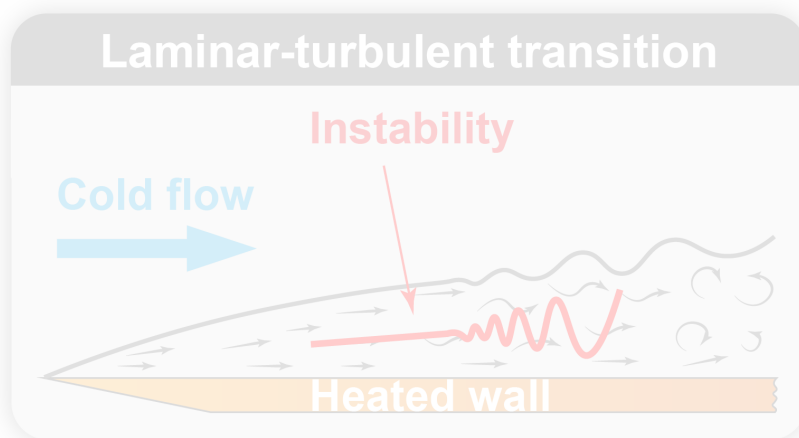


# Transmissivity at high pressures



# How does compressibility affect turbulence?

1. “Universal” scaling laws
2. Turbulence models for RANS (& LES)





# Scaling of turbulent statistics

**Incompressible flow**  
Conventional wall based scaling

$$u_\tau = \sqrt{\tau_w / \rho_w}$$

Friction velocity

$$\delta_v = \mu_w / (\rho_w u_\tau)$$

Viscous length scale

$$Re_\tau = h / \delta_v$$

Friction Reynolds number

$$y^+ = (y/h) Re_\tau$$

Non-dim wall distance

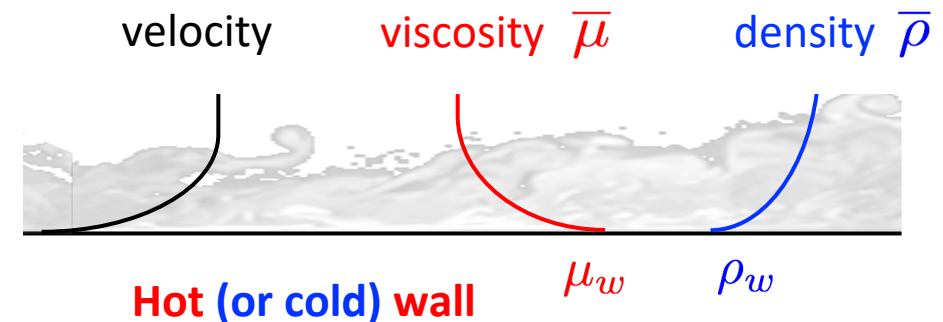
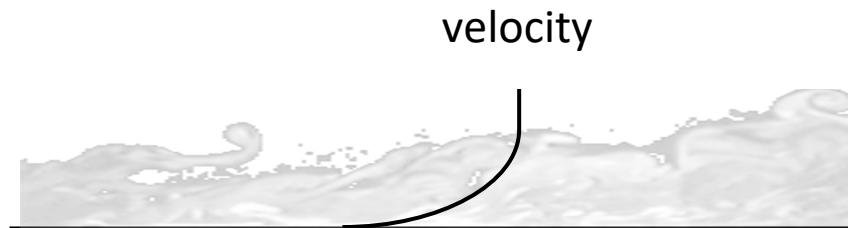
**Compressible flow**  
Semi-local scaling (Huang et al., 1995)

$$u_\tau^* = \sqrt{\tau_w / \bar{\rho}}$$

$$\delta_v^* = \bar{\mu} / (\bar{\rho} u_\tau^*)$$

$$Re_\tau^* = h / \delta_v^* = \sqrt{(\bar{\rho} / \rho_w)} / (\bar{\mu} / \mu_w) Re_\tau$$

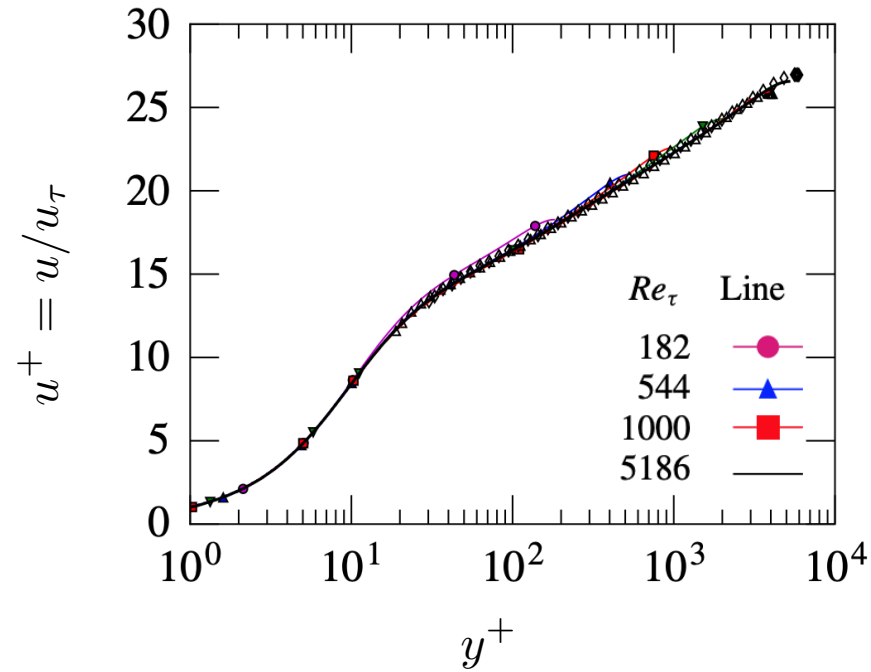
$$y^* = (y/h) Re_\tau^*$$



# Velocity scaling

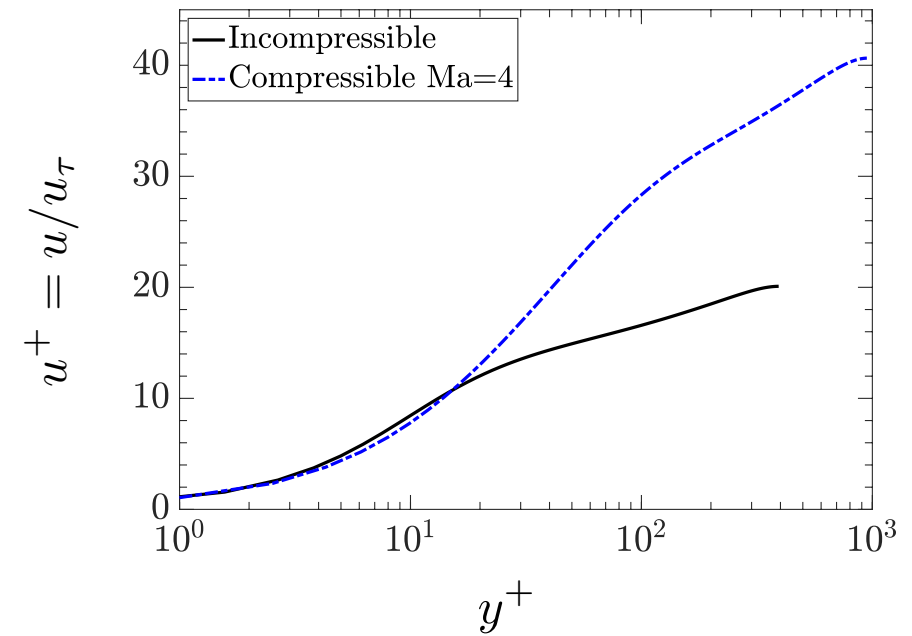
## Incompressible channel

(Lee & Moser, JFM 2015)



## Compressible channel

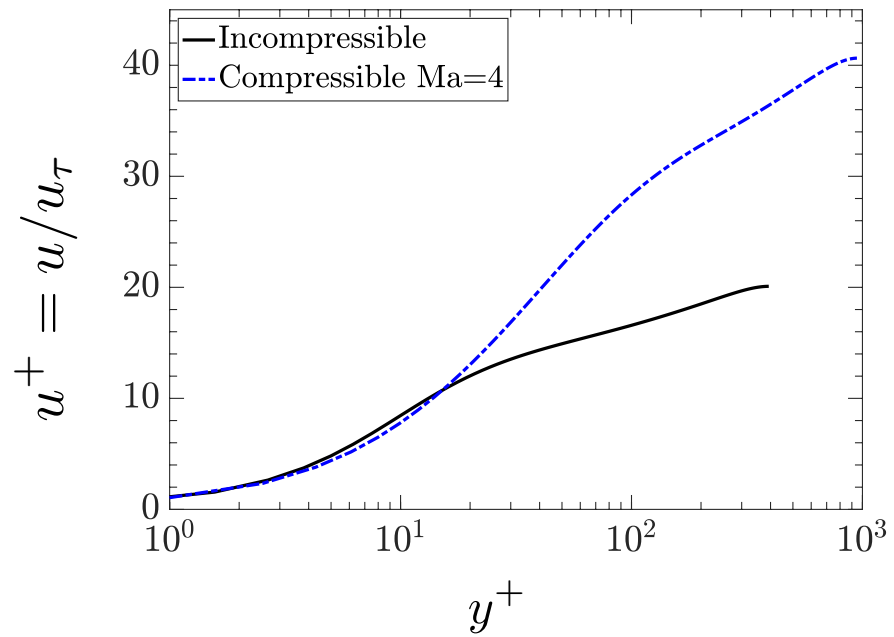
(Own unpublished work)



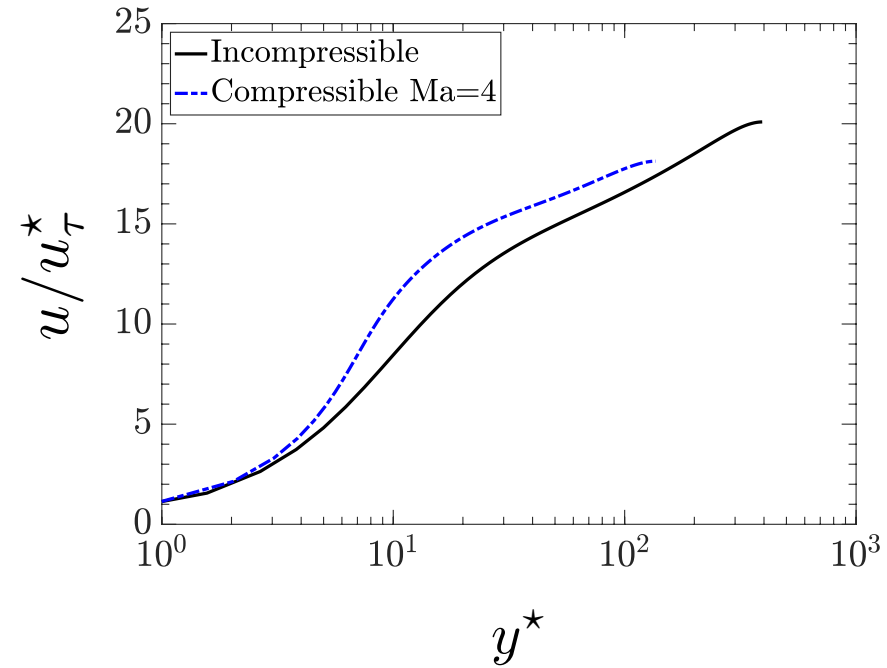
# Velocity scaling

## Compressible channel

(Own unpublished work)



Scaling based on wall units

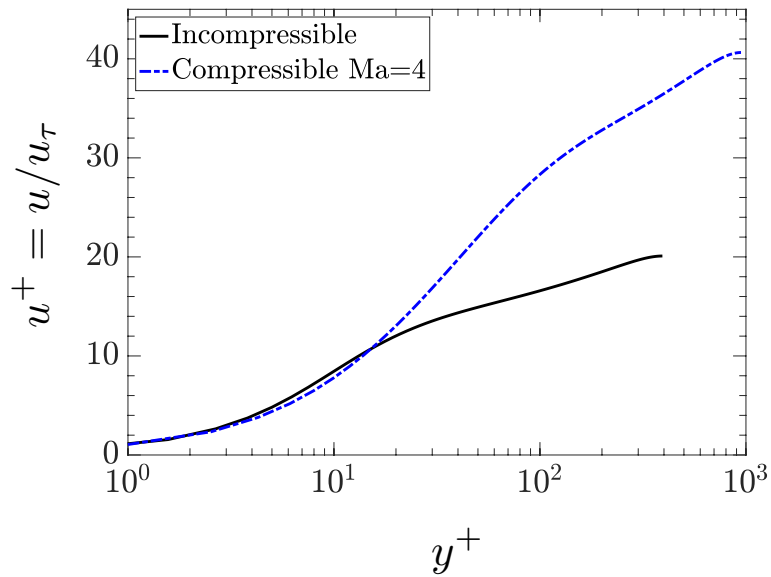


Scaling based on semi-local units

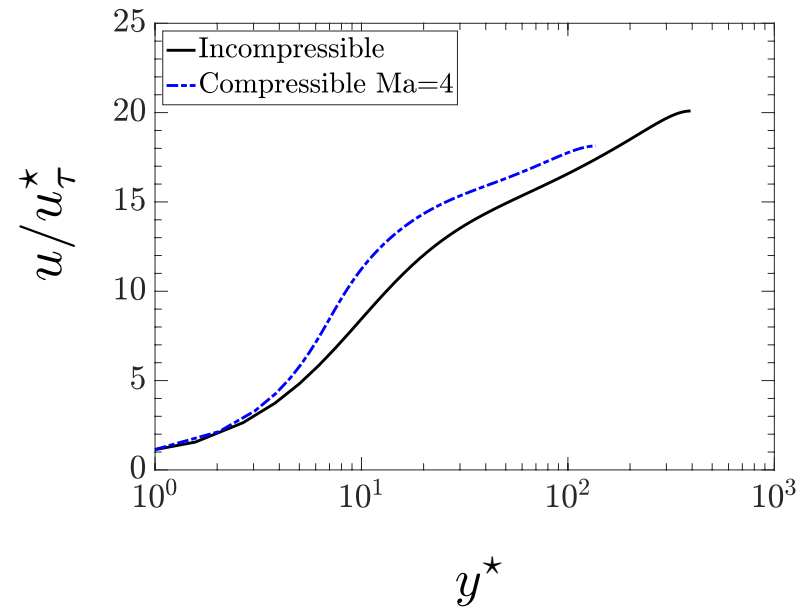
# Velocity scaling

## Compressible channel

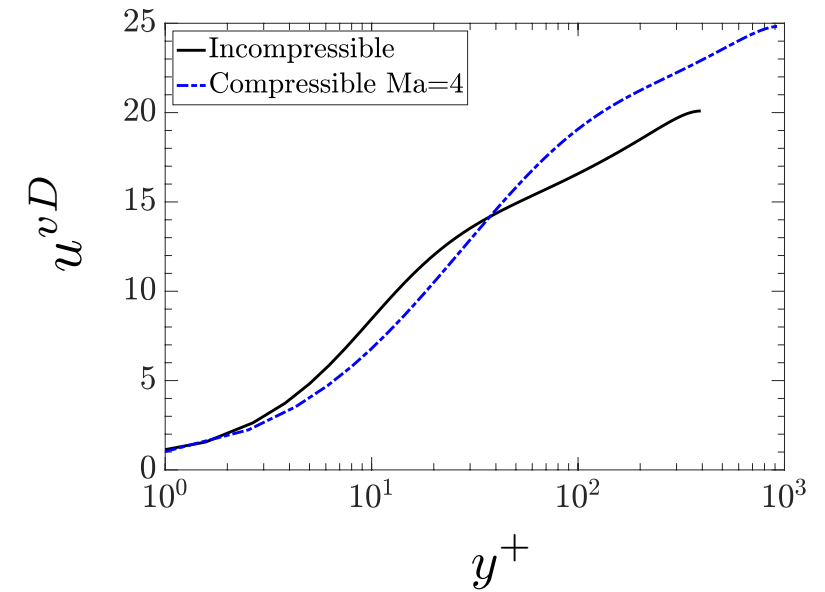
(Own unpublished work)



Scaling based on wall units



Scaling based on semi-local units

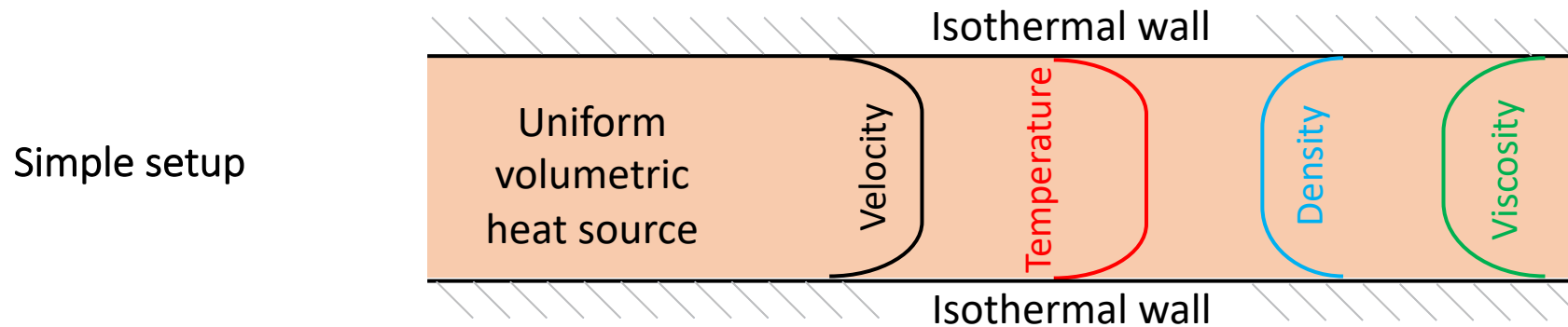


van Driest velocity scaling, (1951)

$$u^{vD} = \int_0^{u^+} \sqrt{\rho/\rho_w} du^+$$

Is it possible to derive a “universal” scaling for compressible flows?

# Turbulent channel flow configuration



## Governing equations

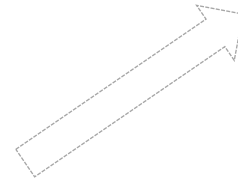
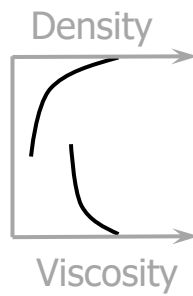
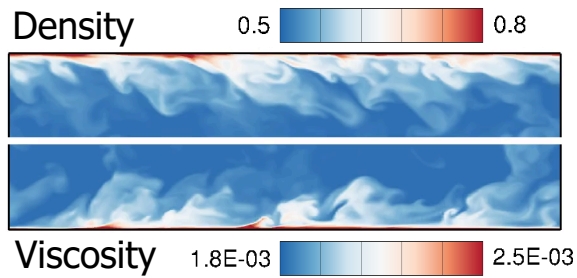
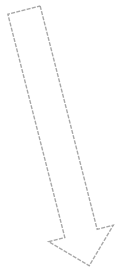
- Continuity  $\partial_t \rho + \partial_{x_j} (\rho u_j) = 0,$
- Momentum  $\partial_t (\rho u_i) + \partial_{x_j} (\rho u_i u_j) = -\partial_{x_i} p + \frac{1}{Re_\tau} \partial_{x_j} (2\mu S_{ij}),$
- Enthalpy  $\partial_t (\rho H) + \partial_{x_j} (\rho u_j H) = \frac{1}{Re_\tau Pr_w} \partial_{x_j} (\lambda \partial_{x_j} T) + \frac{\phi}{Re_\tau Pr}$

## Numerical schemes

- Low-Mach number/anelastic approximation of Navier-Stokes equations (no acoustic waves)
- 6<sup>th</sup> order compact finite difference (staggered) in wall normal direction
- Pseudo spectral method in periodic directions (skew-symmetric form of advective terms)

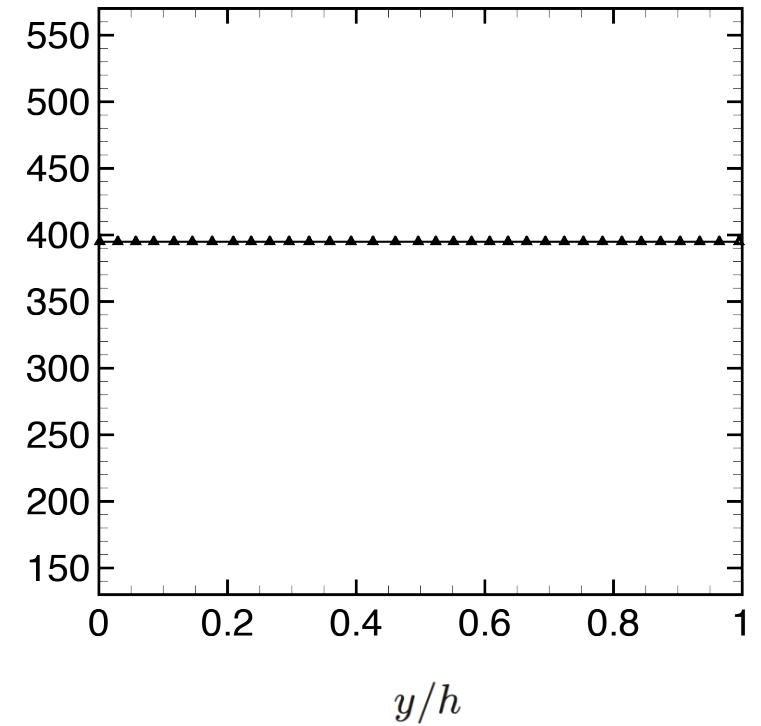
# Volumetrically heated channel flows

Case	Density $\rho/\rho_w$	Viscosity $\mu/\mu_w$
▲ CP395	1	1
— $CRe_\tau^*$	$(T/T_w)^{-1}$	$(T/T_w)^{-0.5}$



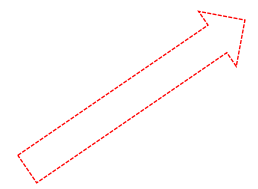
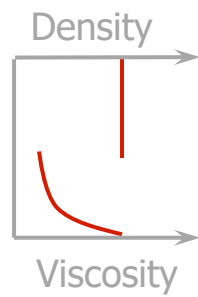
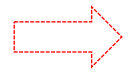
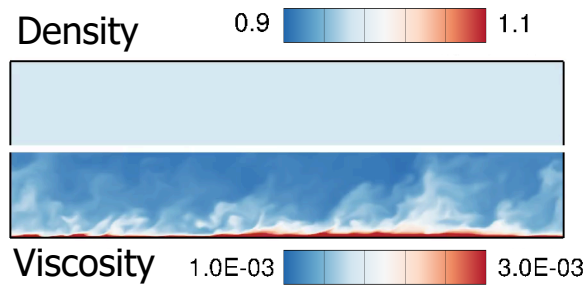
Semi-local Reynolds number

$$Re_\tau^* = \sqrt{(\bar{\rho}/\rho_w)/(\bar{\mu}/\mu_w)} Re_\tau$$



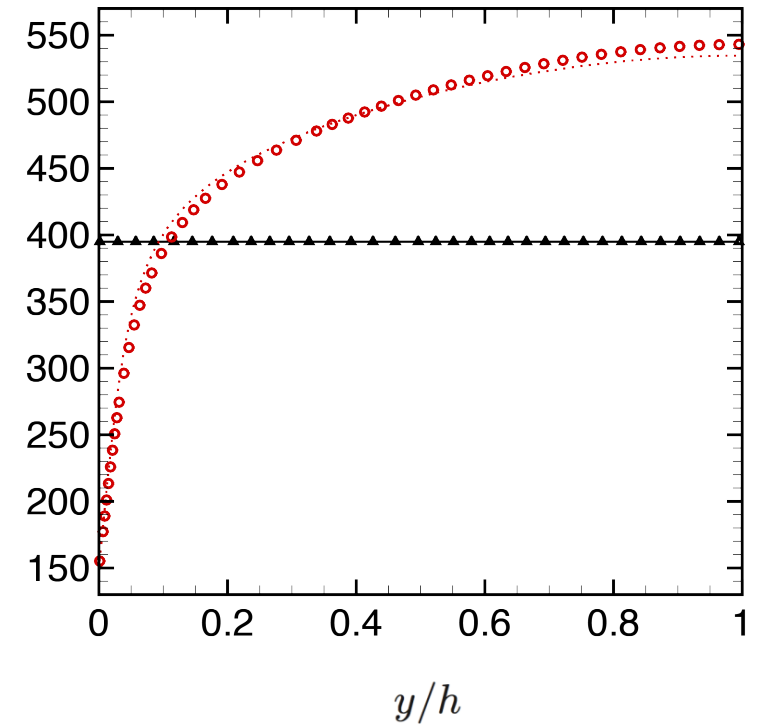
# Volumetrically heated channel flows

Case	Density $\rho/\rho_w$	Viscosity $\mu/\mu_w$
▲ CP395	1	1
— $CRe_\tau^*$	$(T/T_w)^{-1}$	$(T/T_w)^{-0.5}$
● LL	1	$(T/T_w)^{-1}$
⋯ $SRe_{\tau LL}^*$	$(T/T_w)^{0.6}$	$(T/T_w)^{-0.75}$



Semi-local Reynolds number

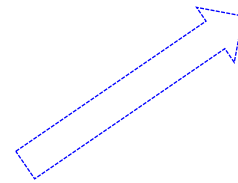
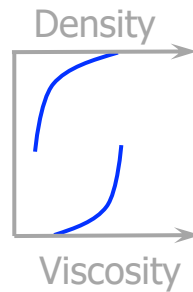
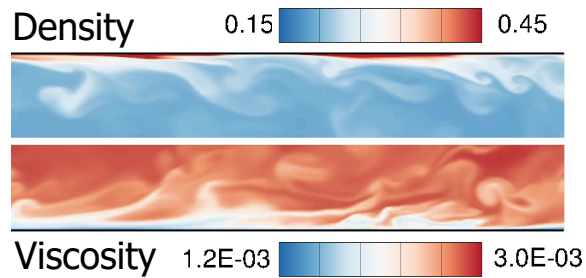
$$Re_\tau^* = \sqrt{(\bar{\rho}/\rho_w)/(\bar{\mu}/\mu_w)} Re_\tau$$





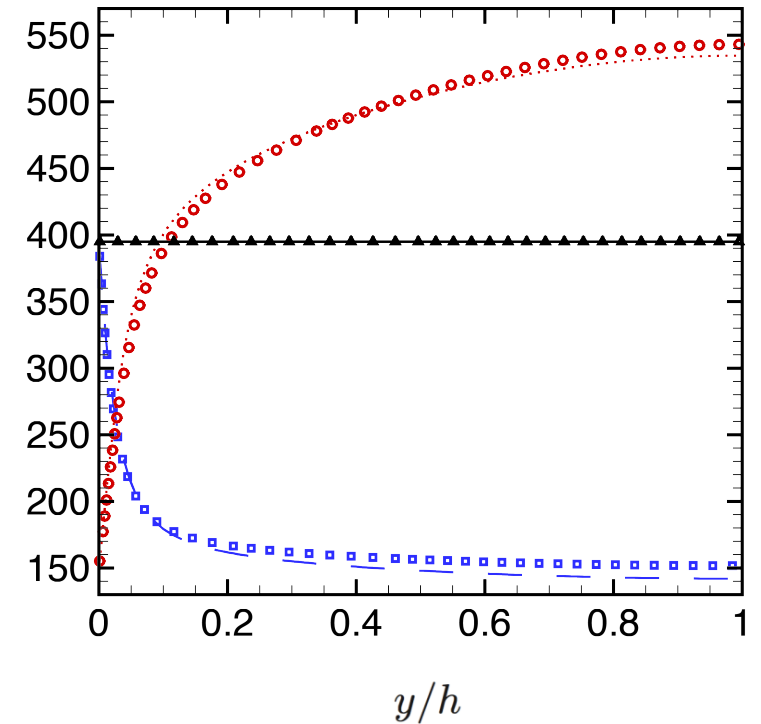
# Volumetrically heated channel flows

Case	Density $\rho/\rho_w$	Viscosity $\mu/\mu_w$
▲ CP395	1	1
— $CRe_\tau^*$	$(T/T_w)^{-1}$	$(T/T_w)^{-0.5}$
● LL	1	$(T/T_w)^{-1}$
⋯ $SRe_{\tau LL}^*$	$(T/T_w)^{0.6}$	$(T/T_w)^{-0.75}$
- - GL	$(T/T_w)^{-1}$	$(T/T_w)^{0.7}$
■ $SRe_{\tau GL}^*$	1	$(T/T_w)^{1.2}$



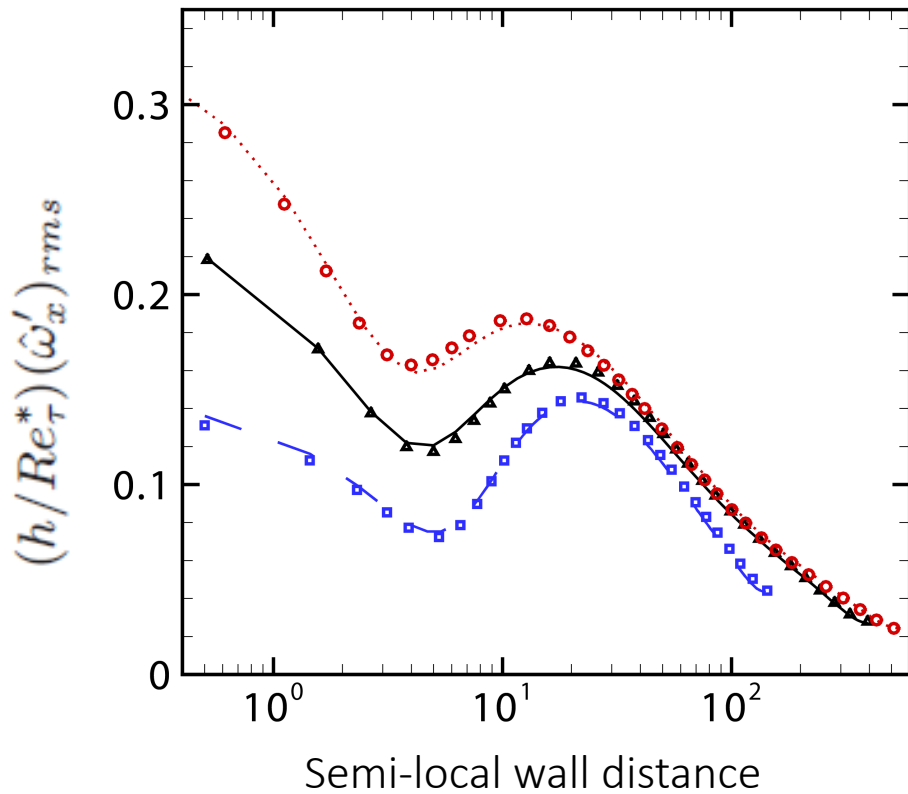
Semi-local Reynolds number

$$Re_\tau^* = \sqrt{(\bar{\rho}/\rho_w)/(\bar{\mu}/\mu_w)} Re_\tau$$



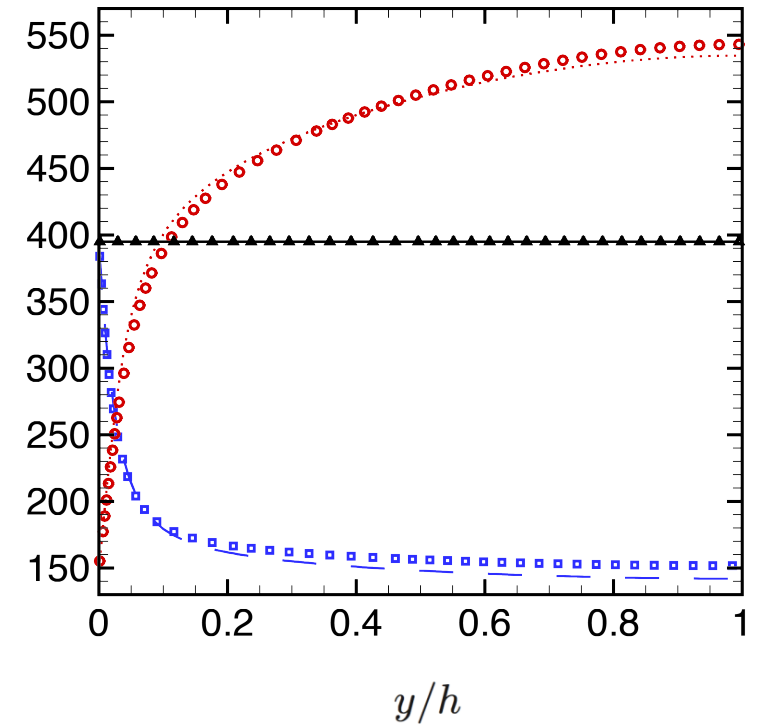
# Turbulent statistics

Streamwise vorticity fluctuations



Semi-local Reynolds number

$$Re_\tau^* = \sqrt{(\bar{\rho}/\rho_w)/(\bar{\mu}/\mu_w)} Re_\tau$$



**Semi-local Reynolds number is governing parameter of turbulence statistics!**

# Extending semi-local scaling framework (Patel et al., PoF 2015)

- Idea: use semi-local scaling transformations for evolution equations

$$\hat{\rho} = \rho/\bar{\rho}, \quad \hat{\mu} = \mu/\bar{\mu}, \quad \hat{u} = u/u_\tau^* \quad \text{with: } u_\tau^* = \sqrt{\tau_w/\bar{\rho}}$$

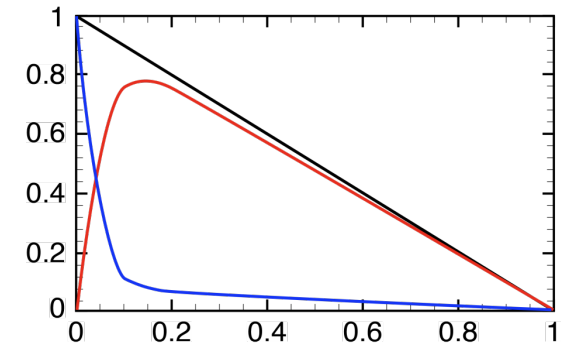
- Momentum:

viscous terms governed by semi-local Reynolds number

$$\hat{\rho} \frac{\partial \hat{u}_i}{\partial \hat{t}} + \hat{\rho} \hat{u}_j \frac{\partial \hat{u}_i}{\partial \hat{x}_j} - \hat{u}_i \hat{\rho} \hat{u}_j C_j = -\frac{\partial \hat{p}}{\partial \hat{x}_i} + \frac{\partial}{\partial \hat{x}_j} \left[ \frac{2\hat{\mu}}{\text{Re}_\tau^*} \left( \hat{S}_{ij} - \hat{D}_{ij} \right) \right]$$

- Stress balance equation (fully dev. turbulent channel)

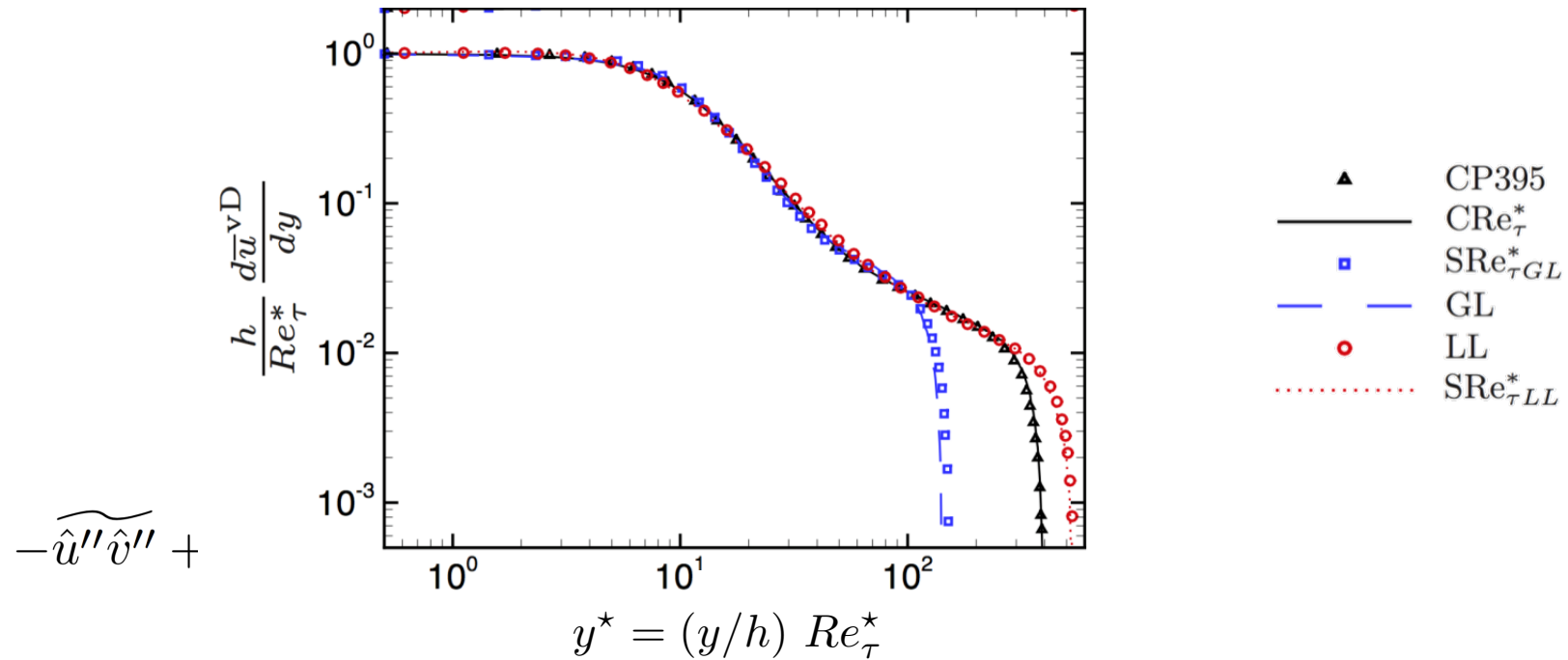
$$\underbrace{-\overline{\hat{u}''\hat{v}''}}_{\text{Turbulent shear stress}} + \underbrace{\frac{h}{\text{Re}_\tau^*} \frac{d\bar{u}^{\text{VD}}}{d\hat{y}}}_{\text{Viscous stress}} \approx \frac{\tau}{\tau_w} = \underbrace{\left(1 - \frac{y}{h}\right)}_{\text{Total stress}}$$



Semi-local Reynolds number is governing parameter of turbulence statistics!

# Semi-locally scaled stress balance equation

$$-\widetilde{\hat{u}''\hat{v}''} + \underbrace{\frac{h}{Re_\tau^*} \frac{d\bar{u}^{vD}}{d\hat{y}}}_{\text{Viscous stress}} \approx \frac{\tau}{\tau_w} = \left(1 - \frac{y}{h}\right)$$



# Semi-locally scaled stress balance equation

$$-\widetilde{\hat{u}''\hat{v}''} + \underbrace{\frac{h}{Re_\tau^*} \frac{d\bar{u}^{vD}}{d\hat{y}}}_{\text{Viscous stress}} \approx \frac{\tau}{\tau_w} = \left(1 - \frac{y}{h}\right)$$

Scaled viscous stress is  
basis for transformation:

$$\frac{h}{Re_\tau^*} \left( \frac{dy^*}{dy} \right) \frac{d\bar{u}^{vD}}{dy^*} = \Phi(y^*)$$

with:

$$y^* = y Re_\tau^* / h$$

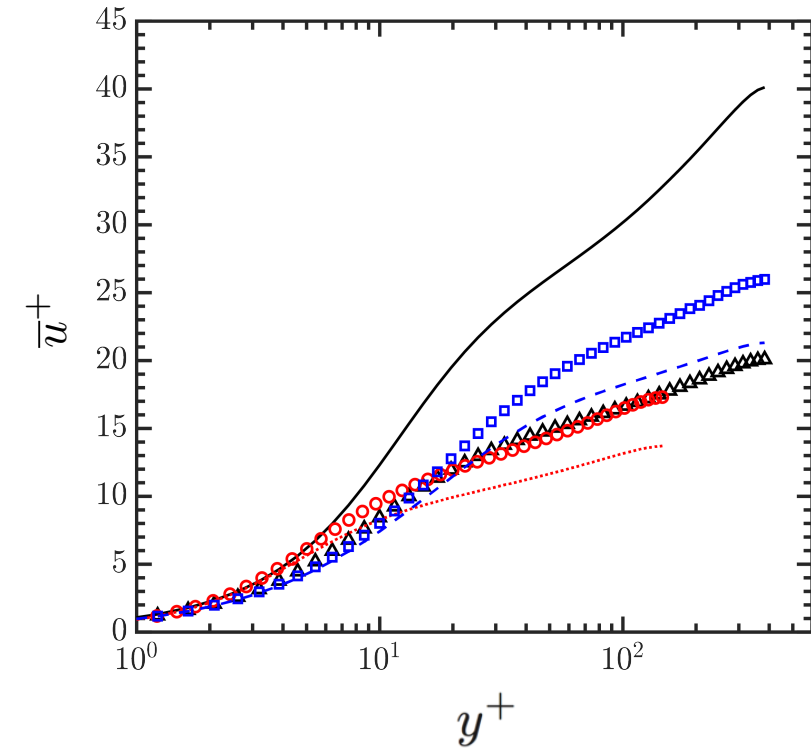
Universal velocity  
transformation:

$$\bar{u}^* = \int_0^{\bar{u}^{vD}} \left( 1 + \frac{y}{Re_\tau^*} \frac{dRe_\tau^*}{dy} \right) d\bar{u}^{vD}$$

# Velocity scaling for channel flows

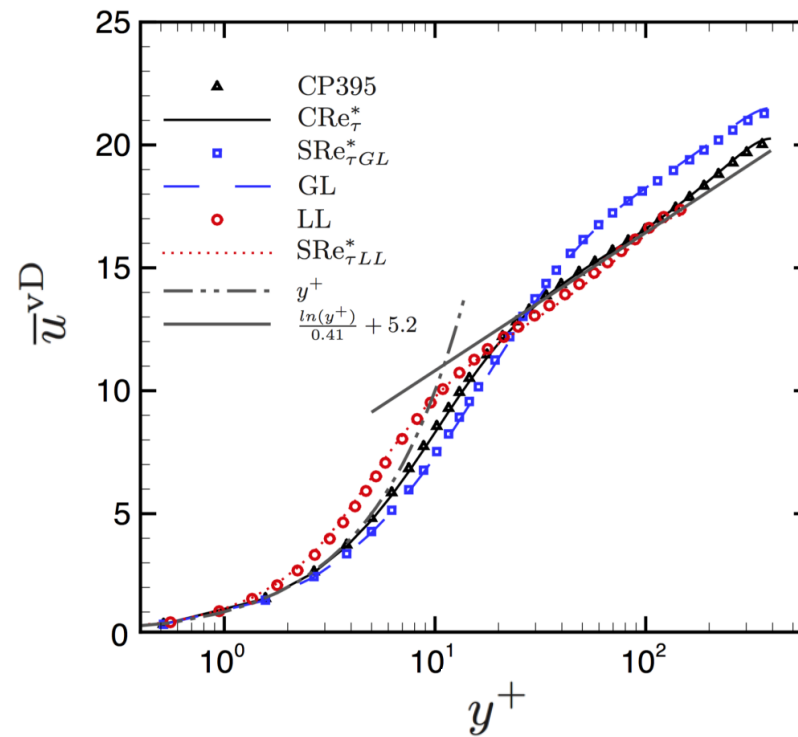
Scaling based on wall units

$$\bar{u}^+ = \bar{u}/u_\tau$$



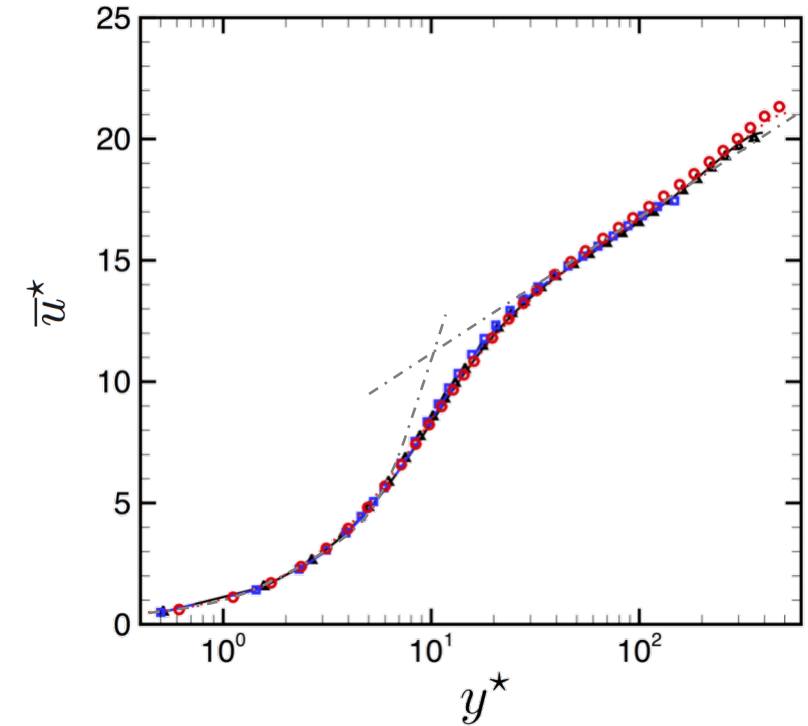
Van Driest transformation

$$\bar{u}^{vD} = \int_0^{\bar{u}} \sqrt{\bar{\rho}/\rho_w} d(\bar{u}/u_\tau)$$



“Universal” transformation

$$\bar{u}^* = \int_0^{\bar{u}^{vD}} \left( 1 + \frac{y}{Re_\tau^*} \frac{dRe_\tau^*}{dy} \right) d\bar{u}^{vD}$$



How do conventional turbulence models perform?

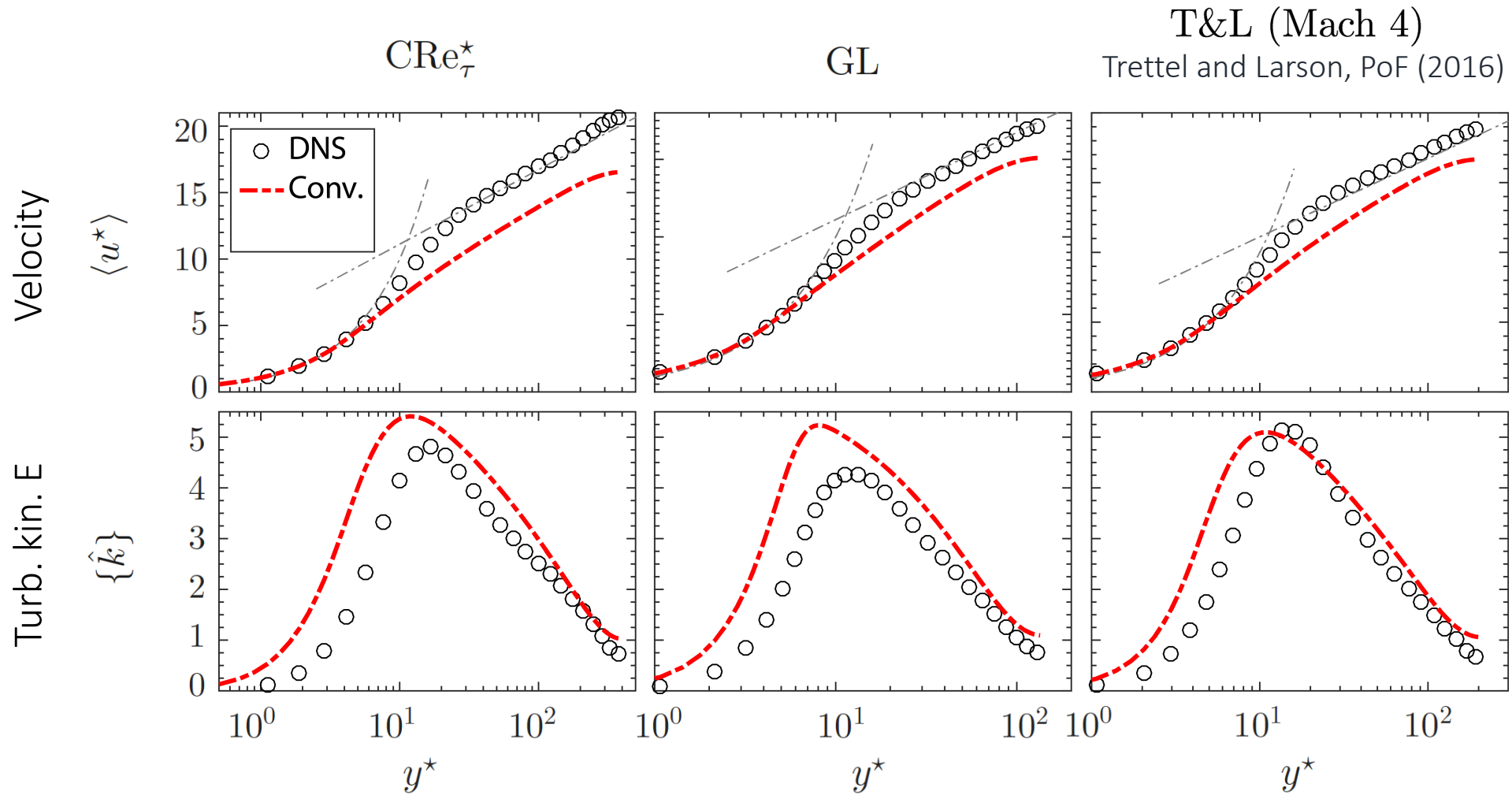
# How do conventional turbulence models perform?

Reynolds/Favre averaged equations for fully developed turbulent channel

- Streamwise momentum  $\frac{\partial}{\partial y} \left[ \left( \frac{\mu}{Re} + \mu_t \right) \frac{\partial u}{\partial y} \right] = -\rho f_x,$
- Energy equation  $\frac{\partial}{\partial y} \left[ \left( \frac{\lambda}{Re Pr_w Ec} + \frac{c_p \mu_t}{Pr_t} \right) \frac{\partial T}{\partial y} \right] = -\Phi,$
- Turb kinetic energy  $-\frac{\partial}{\partial y} \left[ (\bar{\mu} + \mu_t / \sigma_k) \frac{\partial k}{\partial y} \right] = P_k - \bar{\rho} \epsilon$   
+ supporting eqs.  $\vdots$
- Eddy viscosity  $\mu_t = C_\mu f_\mu \bar{\rho} k^2 / \epsilon$



# V2F turbulence model



Nusslet number error:

20.3%

53.2%

52.9%

# Extending semi-local scaling framework

- Using semi-local scaling transformations to non-dimensionalize conservation equations

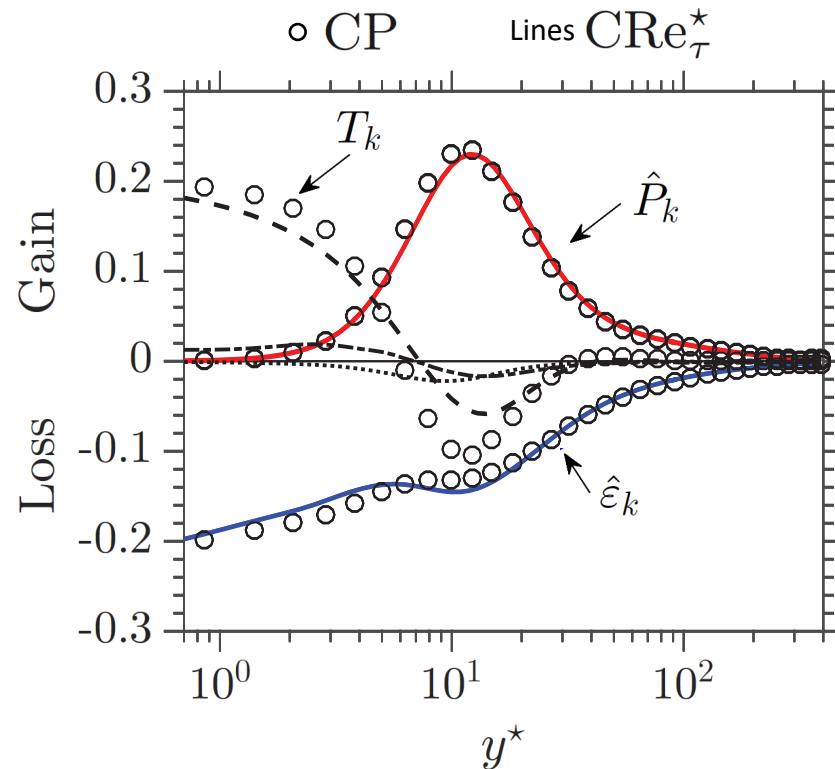
$$\hat{\rho} = \rho / \bar{\rho}, \quad \hat{\mu} = \mu / \bar{\mu}, \quad \hat{u} = u / u_{\tau}^* \quad \text{with: } u_{\tau}^* = \sqrt{\tau_w / \bar{\rho}}$$

- Turbulent kinetic energy

$$\begin{aligned} \frac{\partial \overline{\hat{\rho} \hat{k}}}{\partial \hat{t}} + \frac{\partial \overline{\hat{\rho} \hat{k} \tilde{u}_j}}{\partial \hat{x}_j} = & -P_k - \overline{\hat{\tau}_{ij} \frac{\partial \hat{u}_i''}{\partial \hat{x}_j}} + \frac{\partial}{\partial \hat{x}_j} \left( \overline{\hat{u}_i'' \hat{\tau}_{ij}} - \overline{\hat{\rho} \hat{u}_j'' \frac{1}{2} \hat{u}_i'' \hat{u}_i''} \right) \\ & + \overline{\hat{u}_j'' \frac{\partial \hat{p}}{\partial \hat{x}_j}} + \boxed{\left( \overline{\hat{\rho} \hat{k} \tilde{u}_j} + \overline{\hat{\rho} \hat{u}_j'' \frac{1}{2} \hat{u}_i'' \hat{u}_i''} \right) C_j} \end{aligned}$$

# Semi-locally scaled turbulent kinetic energy budget (Pecnik, JFM 2016)

$$0 = \underbrace{-P_k - \overline{\hat{\tau}_{ij} \frac{\partial \hat{u}_i''}{\partial \hat{x}_j}}}_{\hat{\epsilon}_k} + \underbrace{\frac{\partial}{\partial \hat{x}_j} \left( \overline{\hat{u}_i'' \hat{\tau}_{ij}} - \overline{\hat{\rho} \hat{u}_j'' \frac{1}{2} \hat{u}_i'' \hat{u}_i''} \right)}_{\hat{T}_k} + \underbrace{\overline{\frac{\partial \hat{p}}{\partial \hat{x}_j}}}_{\hat{C}_k} + \underbrace{\left( \overline{\hat{\rho} \hat{k} \hat{u}_i} + \overline{\hat{\rho} \hat{u}_j'' \frac{1}{2} \hat{u}_i'' \hat{u}_i''} \right) C_j}_{\hat{D}_k}$$



# Semi-locally scaled turbulent kinetic energy budget (Pecnik, JFM 2016)

$$0 = -P_k - \underbrace{\overline{\hat{\tau}_{ij} \frac{\partial \hat{u}_i''}{\partial \hat{x}_j}}}_{\hat{\varepsilon}_k} + \underbrace{\frac{\partial}{\partial \hat{x}_j} \left( \overline{\hat{u}_i'' \hat{\tau}_{ij}} - \overline{\hat{\rho} \hat{u}_j'' \frac{1}{2} \hat{u}_i'' \hat{u}_i''} \right)}_{\hat{T}_k} + \underbrace{\overline{\hat{u}_j'' \frac{\partial \hat{p}}{\partial \hat{x}_j}}}_{\hat{C}_k} + \underbrace{\overline{\left( \hat{\rho} \hat{k} \hat{u}_j + \overline{\hat{u}_j'' \frac{1}{2} \hat{u}_i'' \hat{u}_i''} \right) C_j}}_{\hat{D}_k}$$

Fully developed channel



$$-\frac{\partial}{\partial y} \left[ \left( \frac{1}{Re_\tau^*} + \frac{\hat{\mu}_t}{\sigma_k} \right) \frac{\partial \hat{k}}{\partial y} \right] = \hat{\mu}_t \left( \frac{\partial u^{vD}}{\partial y} \right)^2 - \hat{\varepsilon}$$

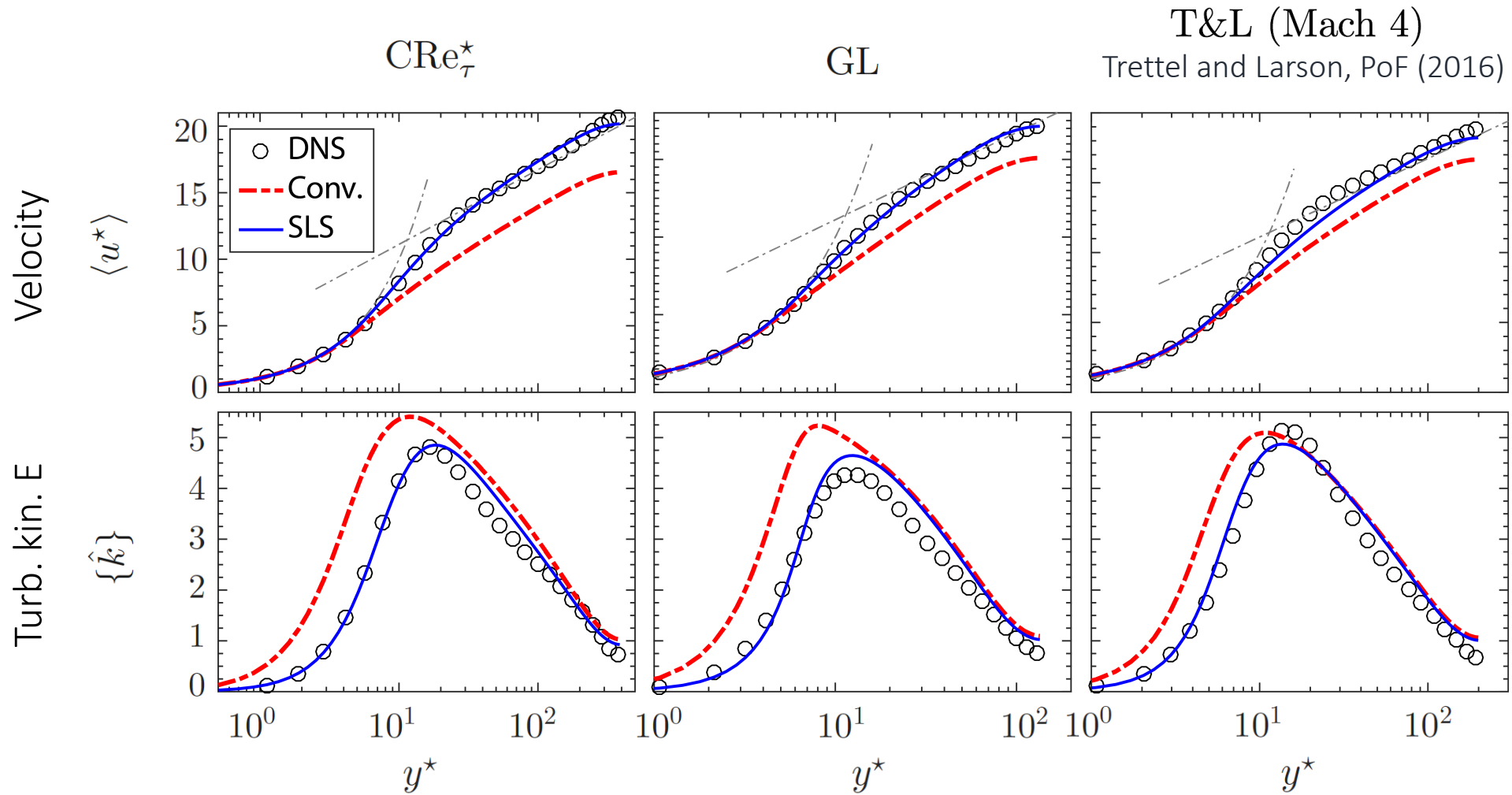
Transforming back to conventional scales



$$\frac{1}{\sqrt{\rho}} \frac{\partial}{\partial x_j} \left[ \frac{1}{\sqrt{\rho}} \left( \frac{\mu}{Re_\tau} + \frac{\mu_t}{\sigma_k} \right) \frac{\partial \rho k}{\partial x_j} \right] = \mu_t \left( \frac{\partial u}{\partial y} \right)^2 - \rho \varepsilon$$

Diffusion of TKE acts upon energy per unit volume !

# V2F turbulence model



# Summary

We proposed:

- Scaling for velocity
- Scaling for temperature
- Generic modification for turbulence models
- Improved results for supercritical flows

# Future directions

- Target modeling buoyancy production term
- Test approach to experimental data at higher Reynolds numbers
- Transition

[https://github.com/Fluid-Dynamics-Of-Energy-Systems-Team/RANS\\_Channel](https://github.com/Fluid-Dynamics-Of-Energy-Systems-Team/RANS_Channel)

Branch: master ▾ New pull request Create new file Upload files Find File Clone or download ▾


Fluid-Dynamics-Of-Energy-Systems-Team Update README.md Latest commit b15e5f8 on 6 Mar

DNS_data	changes	4 months ago
matlab	Update main.m	3 months ago
python	fixed the main.py	2 months ago
.gitignore	cleaned up the directory and added the gitignore	2 months ago
README.md	Update README.md	a month ago
main.ipynb	latex bug fixes	3 months ago
paper.png	picture	4 months ago


README.md

## RANS\_Channel

The source codes available here are based on the [publication](#):



International Journal of Heat and Fluid Flow  
Volume 73, October 2018, Pages 114-123



Turbulence modelling for flows with strong variations in thermo-physical properties

[Gustavo J. Otero R.](#), [Ashish Patel](#), [Rafael Diez S.](#), [Rene Pecnik](#)

The codes solve the Reynolds-averaged Navier-Stokes equations for a fully developed turbulent channel flow with varying properties, such as density and viscosity. The code demonstrates how to modify existing turbulence models to properly account these thermophysical property variations. Five models are used for demonstration:

- an algebraic eddy viscosity model (Cess, 1958),
- the Spalart and Allmaras model (1994),
- k-epsilon model based on Myong and Kasagi (1993),
- Menter's SST k-omega model (Menter, 1995)
- V2F model (Medic and Durbin, 2012).