Modelling and Optimisation of Supercritical Carbon Dioxide Turbomachinery

Ruan van der Westhuizen,
Johan van der Spuy, Albert Groenwold, Robert Dobson

Solar Thermal Energy Research Group (STERG)
Department of Mechanical and Mechatronic Engineering
Stellenbosch University
South Africa

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Preface

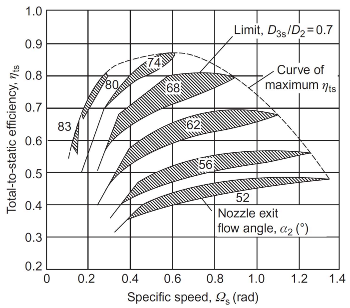
- sCO_2 turbomachinery industry is undeveloped
- Conventional methods of turbomachinery design are not necessarily applicable to sCO_2
- Global optimisation (exploration of full design space) is difficult to achieve using conventional methods
- Turbomachinery design is usually decoupled from system design and optimisation

Contents

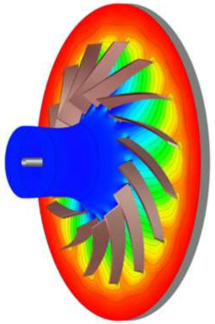
1. Conventional design/optimisation methods
2. Objectives of new design/optimisation method
3. Overview of turbomachinery models
4. Details and implementation of optimisation
5. Key results
6. Preview of future work areas

Turbomachinery Design Methods

- Iteration and evolution
  - Reliance on empirical data
  - Depends on knowledge and experience of designer
- Computational fluid dynamics (CFD)
  - Computationally expensive, but can be improved somewhat with response surface models (RSM's)
  - Practically impossible to use for system-level optimisation



Source: Dixon and Hall (2014)



Source: NUMECA (2019)

Objectives of New Models

- Describe turbomachinery in isolation and as part of a larger power system
- Gradient-based mathematical optimisation
  - Models should consist exclusively of equations
- Modular and adaptable
  - Add more accurate correlations as empirical data become available
  - Flexibility to choose between accuracy and speed

Structure of an Optimisation Problem

Objective Function

$$\text{Minimise } f(\boldsymbol{x}), \text{subject to}$$

Equality Constraints →

$$g_i(\boldsymbol{x}) = 0 \qquad i = 1,2,\dots,m$$

Inequality Constraints →

$$h_j(\boldsymbol{x}) \leq 0 \qquad j = 1,2,\dots,r$$

Upper Bounds

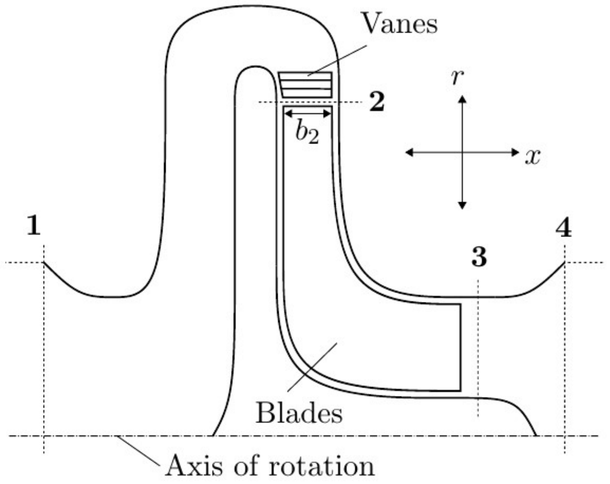
Design Variables →

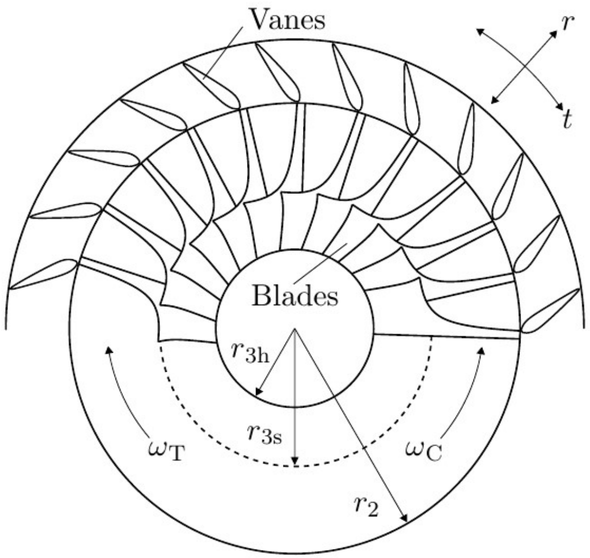
$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \boldsymbol{x} \in \mathbb{R}^n \qquad \check{\boldsymbol{x}} \leq \boldsymbol{x} \leq \hat{\boldsymbol{x}}$$

(Specifications)

Lower Bounds

Geometry of a Radial Turbomachine Stage

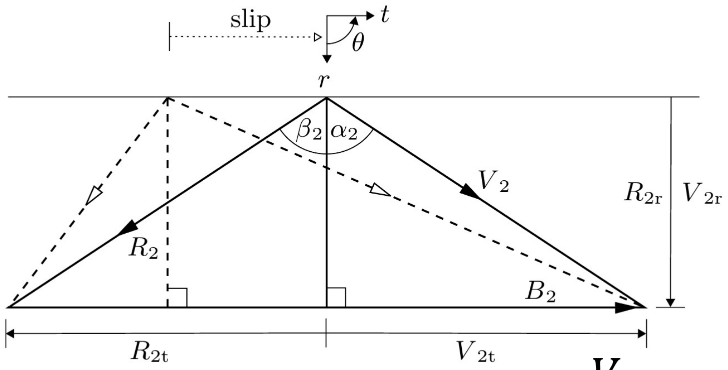




Same notation can be used for
turbine and compressor

Equality Constraints

- Conservation of Mass
  - $\dot{m} = \rho V_m A$
- Conservation of Momentum
  - Velocity triangles



slip factor ⟶ $\sigma = 1 - \frac{V_{2t}}{V_{2t}'}$

- Thermodynamic properties
  - Only two thermodynamic properties are independent
  - Correlations of the form $x_3 = ax_1 + bx_2 + c$ … can be used
- Conservation of Energy and Loss Coefficients

Conservation of Energy: A Closer Look

$\dot{W}_{\text{fluid}} = M\omega$ → “internal” transfer of momentum

$\dot{W}_{\text{thermodynamic}} = \dot{m}(h_{02} - h_{03})$ → “external” measurements

$\dot{W}_{\text{parasitic}} = \left|\dot{W}_{\text{thermodynamic}} - \dot{W}_{\text{fluid}}\right|$ → e.g. clearance, windage

$$\begin{aligned} h_{01} &= h_{02} \\ h_{03} &= h_{04} \end{aligned}$$ → nozzle and diffuser sections

Loss Coefficients

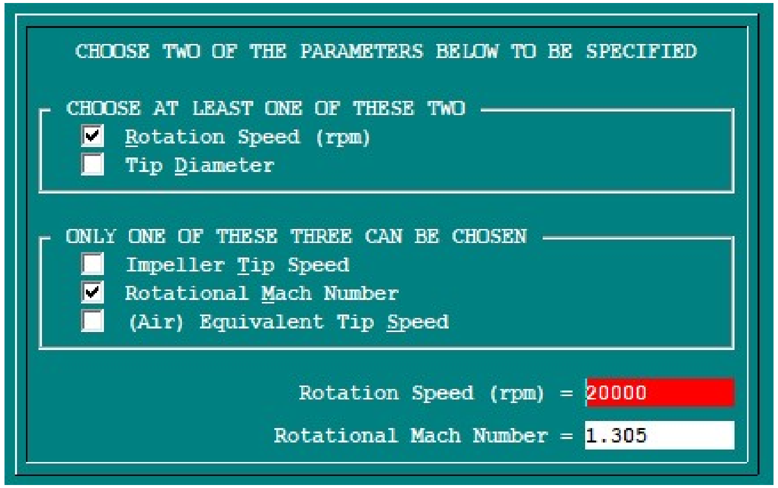
$$\zeta_R = \frac{h_0 - h_{0s}}{\frac{1}{2}(R_2)^2}$$

$$\zeta_P = \frac{w_{\text{parasitic}}}{\frac{1}{2}(R_2)^2}$$

$$\eta_D = \left. \frac{P_{\text{out}} - P_{\text{in}}}{P_{0,\text{in}} - P_{\text{in}}} \middle/ 1 - (AR)^2 \right.$$

- Dimensionless
- Function of machine design
- Function of operating conditions
- Determine numerical values using
  - Experiments
  - CFD

Introduction to CompAero



CHOOSE TWO OF THE PARAMETERS BELOW TO BE SPECIFIED

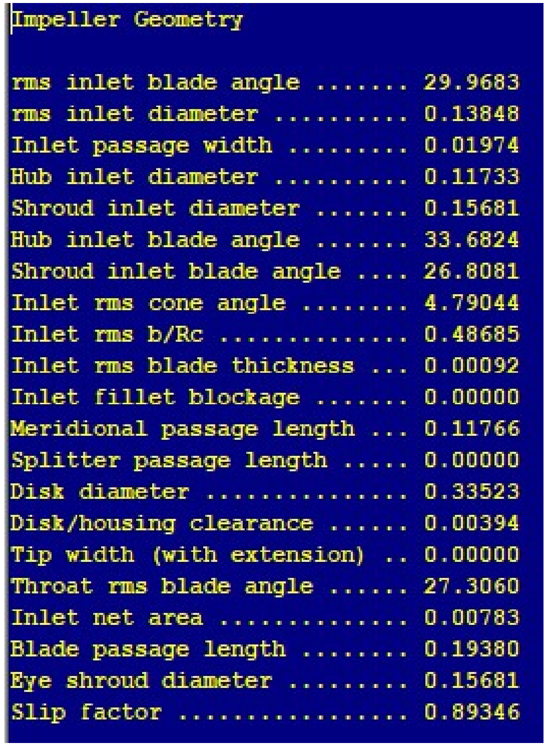
CHOOSE AT LEAST ONE OF THESE TWO
- ☑ Rotation Speed (rpm)
- ☐ Tip Diameter

ONLY ONE OF THESE THREE CAN BE CHOSEN
- ☐ Impeller Tip Speed
- ☑ Rotational Mach Number
- ☐ (Air) Equivalent Tip Speed

Rotation Speed (rpm) = 20000

Rotational Mach Number = 1.305

interface for providing inputs



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Impeller Geometry

rms inlet blade angle ....... 29.9683
rms inlet diameter .......... 0.13848
Inlet passage width ......... 0.01974
Hub inlet diameter .......... 0.11733
Shroud inlet diameter ....... 0.15681
Hub inlet blade angle ....... 33.6824
Shroud inlet blade angle .... 26.8081
Inlet rms cone angle ........ 4.79044
Inlet rms b/Rc .............. 0.48685
Inlet rms blade thickness ... 0.00092
Inlet fillet blockage ....... 0.00000
Meridional passage length ... 0.11766
Splitter passage length ..... 0.00000
Disk diameter ............... 0.33523
Disk/housing clearance ...... 0.00394
Tip width (with extension) .. 0.00000
Throat rms blade angle ...... 27.3060
Inlet net area .............. 0.00783
Blade passage length ........ 0.19380
Eye shroud diameter ......... 0.15681
Slip factor ................. 0.89346
```

results

Calibration and Verification

Calibrated coefficients for zero error with CompAero:

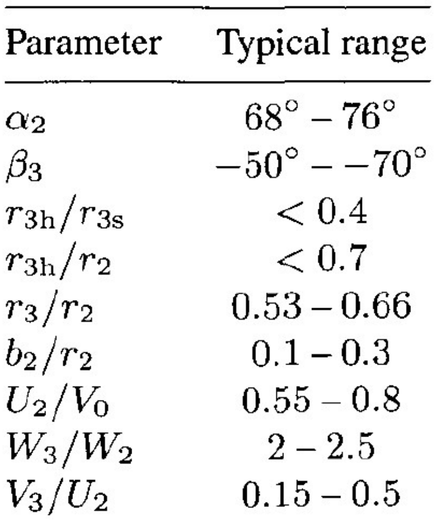
Coefficients	Value
Rotor loss coefficient, ζ_R	0.619
Parasitic work coefficient, ζ_P	0.0108
Diffuser efficiency, η_D	0.972
Slip factor, σ	0.808

Observed errors (same coefficients applied to different machine):

Results	CompAero	This work	Error
Tip total temperature, T_{02}	388 K	385 K	0.8%
Tip total pressure, P_{02}	375 kPa	370 kPa	1.3%
Stage exit total temperature, T_{01}	388 K	384 K	1.0%
Stage exit total pressure, P_{01}	284 kPa	321 kPa	13%
Fluid power, $\dot{W}_F$	28.0 kW	26.3 kW	6.1%

Inequality Constraints

- Mach number limits
- Converging area ratio for nozzle section
- Diverging area ratio for diffuser section
- Typical design ranges (Korpela, 2011) →
  - Angles
  - Velocity ratios
  - Blade geometry ratios



Parameter	Typical range
α_2	$68^\circ - 76^\circ$
β_3	$-50^\circ - -70^\circ$
r_{3h}/r_{3s}	< 0.4
r_{3h}/r_2	< 0.7
r_3/r_2	$0.53 - 0.66$
b_2/r_2	$0.1 - 0.3$
U_2/V_0	$0.55 - 0.8$
W_3/W_2	$2 - 2.5$
V_3/U_2	$0.15 - 0.5$

Practical Implementation

1. Write a function that returns the solution to the inequality and equality constraints: $\boldsymbol{g}, \boldsymbol{h} = \text{function1}(\boldsymbol{x})$
2. Write a function that returns the solution to the objective function: $f = \text{function2}(\boldsymbol{x})$
3. List upper and lower bounds for all variables: $\check{\boldsymbol{x}}$ and $\hat{\boldsymbol{x}}$
4. Select appropriate constraint tolerance (e.g. 0.001 $\approx$ 0)
5. Solve for various random starting points: $\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_N$

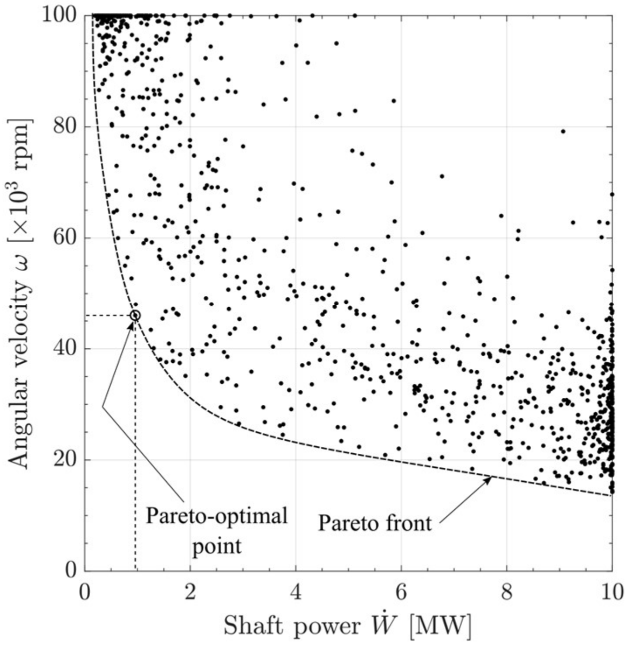
   (*fmincon* in MATLAB Optimization Toolbox or similar algorithm)

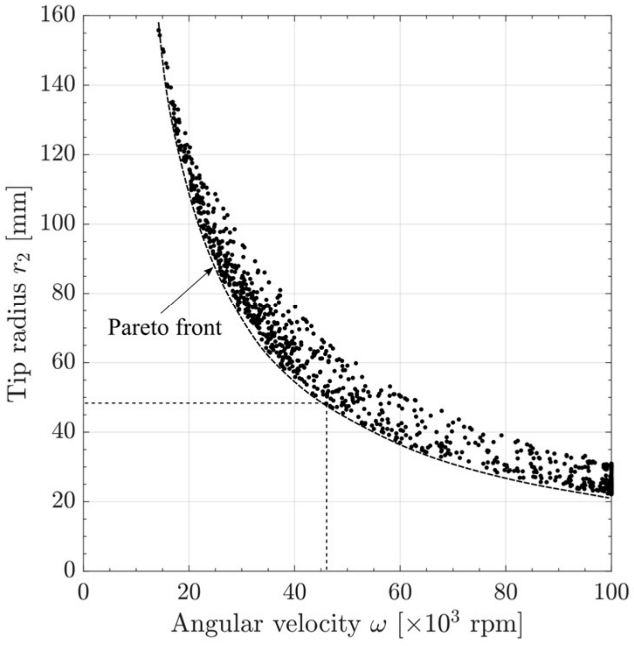
Starting points are independent: speed scales well with CPU single-thread performance **and** CPU count

Compressor Optimisation Results

Key Constraints:

- Constant inlet temperature and pressure
- Constant pressure ratio

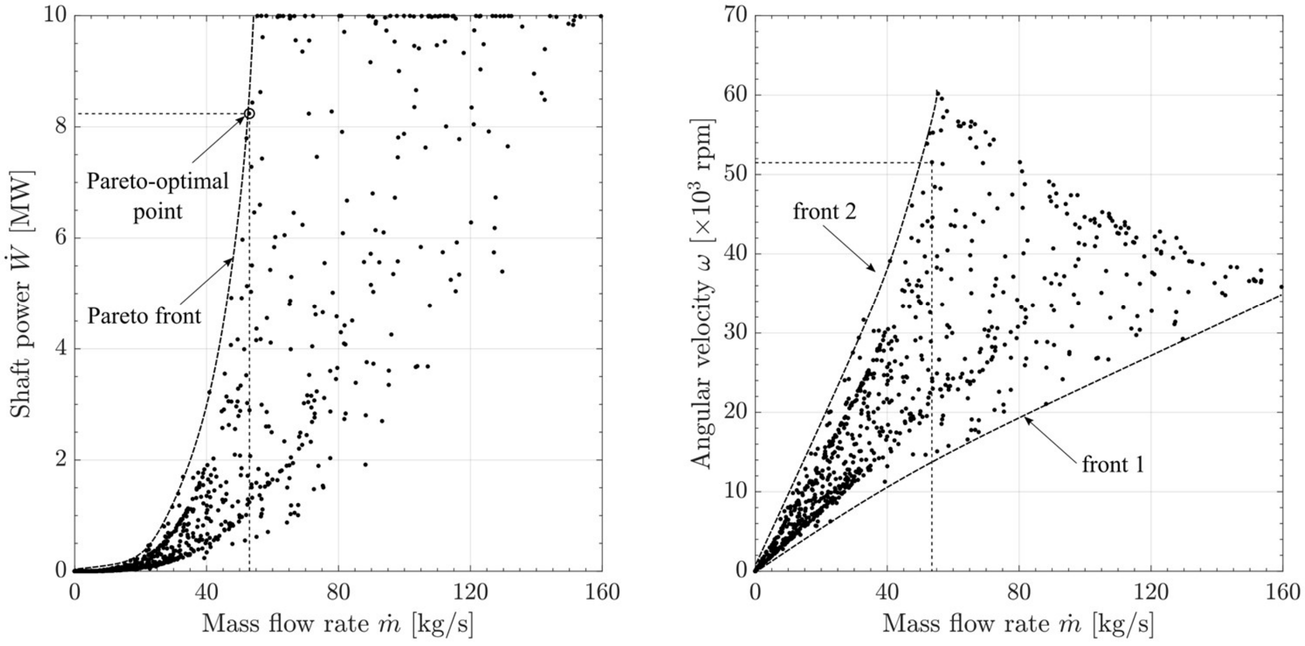




Turbine Optimisation Results

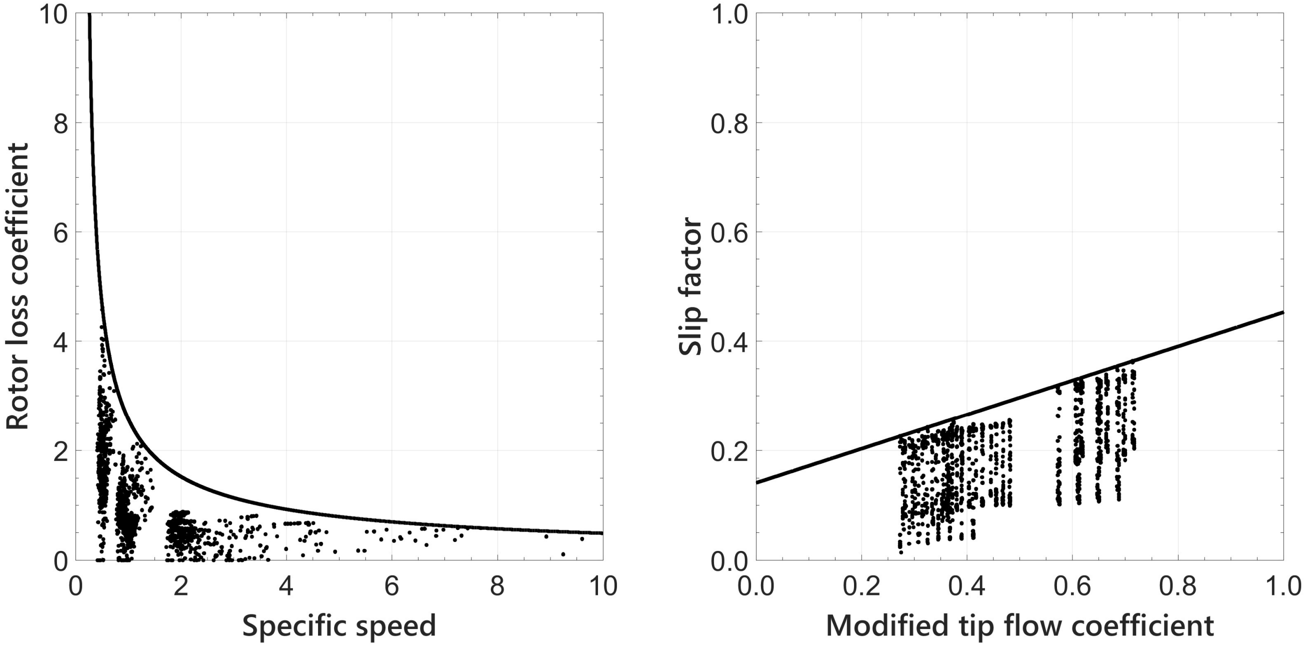
Key Constraints:

- Constant inlet temperature and pressure
- Constant rotor tip radius



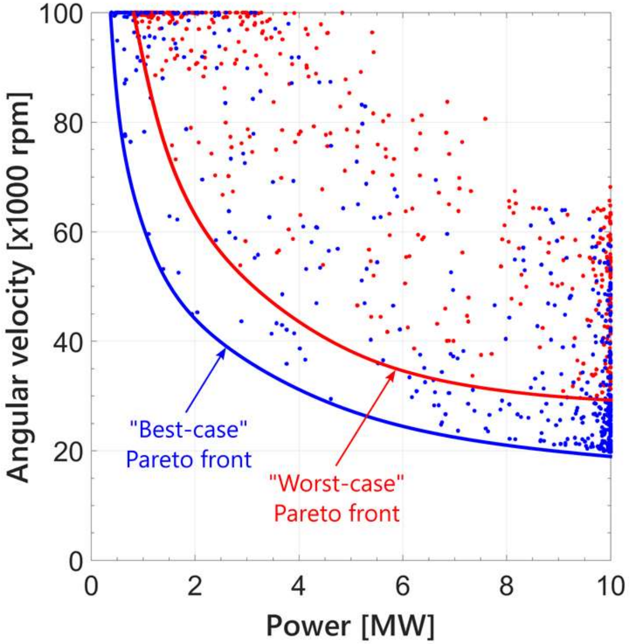
Future Work: Improved Loss Modelling

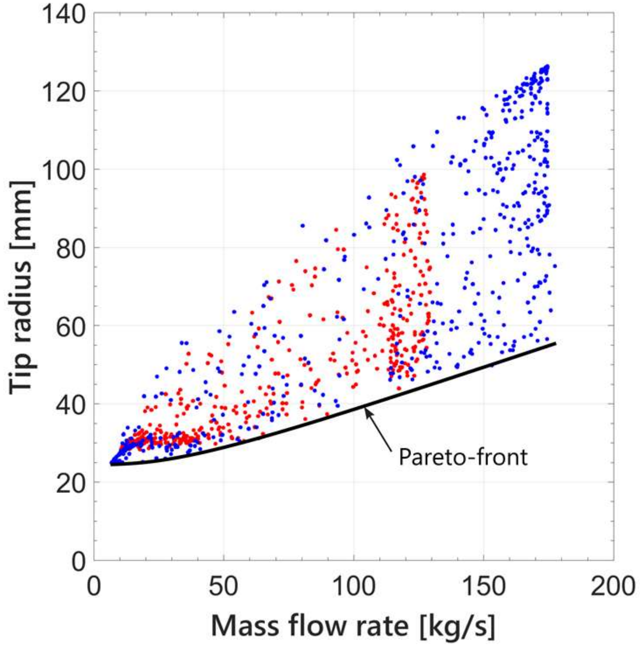
Seek coefficients that are universally applicable
(i.e. correlated against dimensionless parameters)



Future Work: Improved Loss Modelling

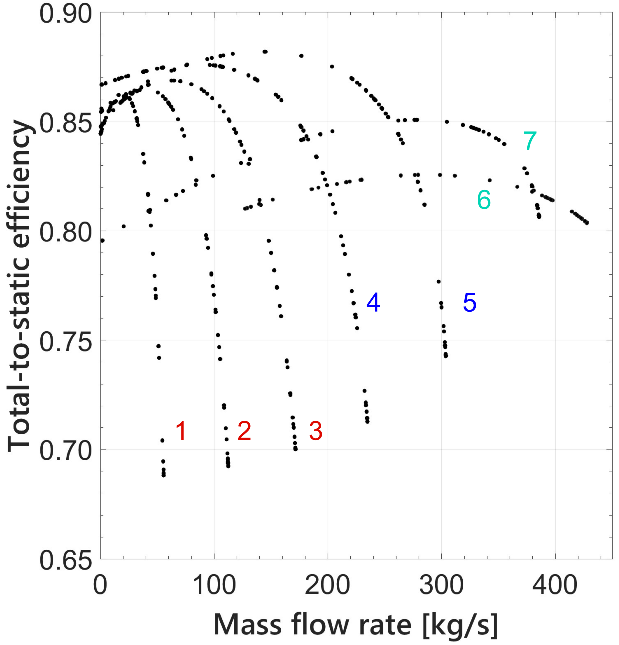
- Pareto front may or may not be influenced by magnitude of losses
- Suggests that optimal values of some variables may be independent of losses

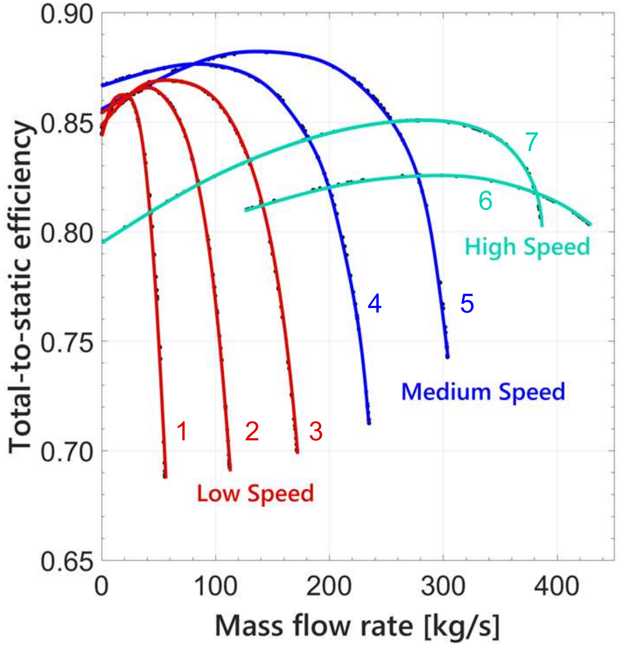




Future Work: Off-Design Performance

- Same optimisation method and algorithm can be applied to generate off-design performance maps
- Requires only changes to which variables are kept constant





Concluding Remarks

- An alternative turbomachinery design method has been introduced
  - Can be applied to system-level modelling
  - Global mathematical optimisation as basis
  - Can easily be extended to off-design performance modelling

- It is useful to choose sensible constraints a priori

- Accurate loss modelling is an important consideration for sCO_2 turbomachinery design,
  but losses only influence some variables

Acknowledgements

Solar Thermal Energy Research Group (STERG)
Stellenbosch University
South Africa

STERG@sun.ac.za
+27 (0)21 808 4016

concentrating.sun.ac.za

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Ruan van der Westhuizen
rvanderwesthuizen@sun.ac.za