



Andreas Jäger, Erik Mickoleit, Cornelia Breitkopf Institute of Power Engineering / Technical Thermodynamics

Accurate and Predictive Mixture Models Applied to Mixtures with CO₂

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TECHNIK ÜBERWINDET GRENZEN

Outline

Introduction and Motivation

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Summary and Perspective







Introduction and Motivation

- The efficiency of processes with sCO_2 can be potentially optimized by using blends
- Example: Shift of critical point









Introduction and Motivation

- The efficiency of processes with sCO_2 can be potentially optimized by using blends
- Example: Shift of critical point











Introduction and Motivation

- Multiparameter equations of state (e.g. Span and Wagner (1996)) can be used in a multifluid mixture model (Lemmon and Tillner-Roth (1999), Kunz and Wagner (2012))
- For accurate models, usually many parameters need to be fitted to experimental data
- Standard mixing rules for predictive calculations do not always give good results
- Recently, we proposed a predictive model, which yields good results for phase equilibria
- Representation of homogeneous properties is studied in this work
- Here: CO_2 + CH_4 and CO_2 + C_2H_6 studied







The multi-fluid mixture model can be written as follows

$$\frac{a}{RT} = \alpha(\tau, \delta, \bar{x}) = \sum_{i=1}^{N} x_i \underbrace{\alpha_{0,i}^{0}(T, \rho)}_{\text{Pure}} + \underbrace{\sum_{i=1}^{N} x_i \ln(x_i)}_{\text{Ideal mixing}} + \begin{bmatrix} \text{Mixture of ideal gases:} \\ \text{No adjustable parameters} \end{bmatrix}$$

$$\sum_{i=1}^{N} x_i \underbrace{\alpha_{0,i}^{r}(\tau, \delta)}_{\text{Pure}} + \underbrace{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} x_i x_j F_{ij} \alpha_{ij}^{r}(\tau, \delta)}_{\text{Departure function}} \end{bmatrix}$$

$$\begin{array}{l} \text{Residual part:} \\ \text{Arbitrary number of adjustable parameters} \\ \alpha_{ij}^{r} = \sum_{k=1}^{K_{\text{Pol},ij}} n_{ij,k} \delta^{d_{ij,k}} \tau^{(t_{ij,k})} + \alpha_{ij,\text{EXP}}^{r} + \cdots \end{array}$$







The multi-fluid mixture model can be written as follows

$$\frac{a}{RT} = \alpha(\tau, \delta, \bar{x}) = \sum_{i=1}^{N} x_i \underbrace{\alpha_{0,i}^0(T, \rho)}_{\text{Pure substance}} + \underbrace{\sum_{i=1}^{N} x_i \ln(x_i) + }_{\text{Ideal mixing}} + \underbrace{\sum_{i=1}^{N} x_i \alpha_{0,i}^r(\tau, \delta)}_{\text{Substance}} + \underbrace{\sum_{i=1}^{N} \sum_{j=i+1}^{N} x_i x_j F_{ij} \alpha_{ij}^r(\tau, \delta)}_{\text{Departure function}} + \underbrace{\alpha_{ij}^r = \sum_{k=1}^{K_{\text{Pol},ij}} \alpha_{ij,k} \delta^{(d_{ij,k})} \tau^{(t_{ij,k})} + \alpha_{ij,\text{EXP}}^r + \cdots$$







The reducing parameters are a function of the composition

$$\tau = \frac{T_{\text{red}}(\bar{x})}{T} \qquad T_{\text{red}} = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \beta_{T,ij} \gamma_{T,ij} \frac{(x_i + x_j)}{\beta_{T,ij}^2 x_i + x_j} (T_{\text{c},i} T_{\text{c},j})^{0.5}$$
$$\delta = \frac{\rho}{\rho_{\text{red}}(\bar{x})} \qquad \frac{1}{\rho_{\text{red}}} = v_{\text{red}} = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \beta_{v,ij} \gamma_{v,ij} \frac{(x_i + x_j)}{\beta_{v,ij}^2 x_i + x_j} \frac{1}{8} \left(\frac{1}{\rho_{\text{c},i}^{1/3}} + \frac{1}{\rho_{\text{c},j}^{1/3}}\right)^3$$







If no experimental data are available

→ Standard mixing rules for reducing parameters

Linear mixing rules

$$\beta_{T} = 1 ; \ \beta_{v} = 1 ; \ \gamma_{T} = \frac{1}{2} \frac{\left(T_{c,i} + T_{c,j}\right)}{\left(T_{c,i} \cdot T_{c,j}\right)^{0,5}}; \ \gamma_{v} = 4 \frac{\left(v_{c,i} + v_{c,j}\right)}{\left(v_{c,i}^{1/3} + v_{c,j}^{1/3}\right)^{3}}$$

$$\Rightarrow T_{red} = \sum_{i=1}^{N} x_{i} T_{c,i} \text{ and } v_{red} = \sum_{i=1}^{N} x_{i} v_{c,i}$$

No departure function $\alpha_{ij}^{r} = 0$







The multi-fluid mixture model can be written as follows

$$\frac{a}{RT} = \alpha(\tau, \delta, \bar{x}) = \sum_{i=1}^{N} x_i \underbrace{\alpha_{o,i}^{o}(T, \rho)}_{\text{Pure}} + \underbrace{\sum_{i=1}^{N} x_i \ln(x_i)}_{\text{Ideal mixing}} + \sum_{i=1}^{N} x_i \underbrace{\alpha_{o,i}^{r}(\tau, \delta)}_{\text{Ideal mixing}} + \sum_{i=1}^{N} x_i \underbrace{\alpha_{o,i}^{r}(\tau, \delta)}_{\text{Dependential}} + \underbrace{\alpha_{o,i}^{\text{DEP}}}_{\text{Dependential}}$$

$$\sum_{i=1}^{N} x_i \underbrace{\alpha_{0,i}^{r}(\tau, \delta)}_{\text{Pure substance}} + \underbrace{\alpha_{0,i}^{\text{DEP}}}_{\text{Departure function}}$$







The theoretically based departure function reads (Jäger et al. (2018a, 2018b))









The theoretically based departure function reads (Jäger et al. (2018a, 2018b))

$$\alpha^{\text{Dep}} = \frac{\ln(1+b\rho)}{\ln(1+b\rho_{\text{ref}})} \left\{ \begin{array}{l} g_{\text{GE}}^{\text{E},r} \\ RT \end{array} - \sum_{i=1}^{N} x_i \left[\alpha_{\text{oi}}^{r} (\delta_{\text{ref}}, \tau) - \alpha_{\text{oi}}^{r} (\delta_{i,\text{ref}}, \tau_{i}) \right] \right\}$$
COSMO-SAC
Lin and Sandler (2002) (COSMO-SAC 1)
Hsieh et al. (2010) (COSMO-SAC 2)
Hsieh et al. (2014) (COSMO-SAC 3)
$$g^{\text{E},r} = RT \sum_{i=1}^{N} x_i \ln(\gamma_i^{r})$$
COSMO-SAC 3)







Thermophysical Property Software (TREND)

— All models are implemented in a user-friedly software tool (TREND 4.0)

INPUT PARAMETERS					FLASH CALCULA	TION						
Path to EOS	D:\Arbeit\Stof	datensoft	wares\TREN	D_d			vap	liq1	liq2	sol	hyd	Overall
					temperature	к	253,000		253,000			253,000
Input code	tp				pressure	MPa	1,850		1,850			1,850
Property 1	253	к			density	mol/m³	1156,908		15243,80			4819,935
Property 2	1,85	MPa			int. energy	j/mol	15520,343		6268,134807			7911,433
Unit	molar				enthalpy	j/mol	17119,433		6389,496			8295,256
					entropy	j/(mol K)	76,803		32,374			40,265
Fluids	mole fractions	Eq. Type	Mix. Rules		gibbs energy	j/mol	-2311,644		-1801,074			-1891,757
co2	0,20	1	1 1		helmholtz energy	j/mol	-3910,735		-1922,435			-2275,580
ethane	0,80	1	L		isob. heat capacity	j/(mol K)	68,094		95,446			-6666,000
					isoch. heat capacity	j/(mol K)	41,349		45,321			-6666,000
					speed of sound	m/s	231,397		684,036			-6666,000
					hydration number	-						
					hydrate structure	-						
				ove	rall small cage occup	-						
			0		erall large cage occup	-						
					molecular weight	kg/mol	0,034		0,033			0,033
					phase fraction	mol/mol	0,177611491		0,82238851			
					x1	mol/mol	0,310125		0,176216			co2
					x2	mol/mol	0,689875		0,823783706			ethane









Results Phase Equilibria CO₂+C₂H₆

- Other models typically yield better results for phase equilibria than LIN-MOD
- COS3-MOD:
 - Adjusted dispersion parameter

 $\frac{\varepsilon_{\rm CO_2}}{k_{\rm B}} = 85 \text{ K}$









Results Phase Equilibria CO₂+C₂H₆

- More results for phase equilibria can be found in our recent works









Results Homogeneos Densities CO₂+CH₄

- Experimental data by
 Jaeschke et al. (1991, 1997)
- Uncertainty of the data approx.
 0.1%
- Tendency visible that predictive
 models UNI-MOD, COS1-MOD,
 COS2-MOD, COS3-MOD yield
 better results than LIN-MOD











Results Homogeneos Densities CO₂+C₂H₆



 Tendency visible that predictive models UNI-MOD, COS1-MOD,
 COS2-MOD, COS3-MOD yield better results than LIN-MOD







Summary

- Predictive multi-fluid mixture models and reference models have been compared to experimental data
- Investigated mixtures: CO_2+CH_4 and $CO_2+C_2H_6$
- Results for phase equilibria are better for UNI-MOD and COS-MOD than with LIN-MOD
- Unexpectedly, homogeneous densities are also better with UNI-MOD and COS-MOD than with LIN-MOD
- Hence, the combination of the multi-fluid mixture model with g^{E} -models seems to be better suited for screening purposed than LIN-MOD







Perspective

- Further comparisons needed:
 - Other mixtures
 - More properties (speed of sound, heat capacity, etc.)
- The effect of refitting (general) parameters of UNIFAC and COSMO-SAC should be studied
- The developed models can be used for screenings for promising mixtures
- Further development of these models: TU Dresden, suCOO-Lab







Thank you for your attention! Questions?











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Basic idea: equate g^{E} or a^{E} at a specified reference pressure

 $g_{\text{EOS}}^{\text{E}}(p_0) = g_{\text{GE}}^{\text{E}}$ $a_{\text{EOS}}^{\text{E}}(p_0) = a_{\text{GE}}^{\text{E}}$

Needed: expression for a^{E} from multi-fluid mixture model









Molar Helmholtz energy of the mixture calculated with the multi-fluid mixture model

$$\frac{a(T, p, \bar{x})}{RT} = \alpha(\tau, \delta, \bar{x}) = \sum_{i=1}^{N} x_i \alpha_{o,i}^{o}(T, \rho) + \sum_{i=1}^{N} x_i \ln(x_i) + \sum_{i=1}^{N} x_i \alpha_{o,i}^{r}(\tau, \delta) + \alpha^{\text{Dep}}(T, p, \bar{x})$$

The Helmholtz energy of the mixture needs to be evaluated at T, p und \overline{x}

$$\rho = \rho(T, p, \bar{x}) \rightarrow \delta = \frac{\rho(T, p, \bar{x})}{\rho_{\text{red}}(\bar{x})}$$
$$\tau = \frac{T_{\text{red}}(\bar{x})}{T}$$







Molar Helmholtz energy of the pure components

$$\frac{a_{\mathrm{o}i}(T,p)}{RT} = \alpha_{\mathrm{o},i}^{\mathrm{o}}(T,\rho_i) + \alpha_{\mathrm{o},i}^{\mathrm{r}}(\tau_i,\delta_i)$$

The Helmholtz energies need to be evaluated at the same T and p:

$$\rho_{i} = \rho_{i}(T, p) \rightarrow \delta_{i} = \frac{\rho_{i}(T, p)}{\rho_{c,i}}$$
$$\tau_{i} = \frac{T_{c,i}}{T}$$







The molar excess Helmholtz energy then becomes

$$\frac{a^{\mathrm{E}}(T, p, \bar{x})}{RT} = \sum_{i=1}^{N} x_i [\alpha_{0,i}^{\mathrm{o}}(T, \rho) - \alpha_{0,i}^{\mathrm{o}}(T, \rho_i)] + \sum_{i=1}^{N} x_i [\alpha_{0,i}^{\mathrm{r}}(\tau, \delta) - \alpha_{0,i}^{\mathrm{r}}(\tau_i, \delta_i)] + \alpha^{\mathrm{Dep}}(T, p, \bar{x})$$
With
$$\alpha_{0,i}^{\mathrm{o}}(T, \rho) = f_i(T) + \ln\left(\frac{\rho}{\rho_{\mathrm{c},i}}\right) \quad \text{and} \quad \alpha_{0,i}^{\mathrm{o}}(T, \rho_i) = f_i(T) + \ln\left(\frac{\rho_i}{\rho_{\mathrm{c},i}}\right)$$
it is

$$\frac{a^{\mathrm{E}}(T,p,\bar{x})}{RT} = \sum_{i=1}^{N} x_i \ln\left(\frac{\rho}{\rho_i}\right) + \sum_{i=1}^{N} x_i [\alpha_{\mathrm{o},i}^{\mathrm{r}}(\tau,\delta) - \alpha_{\mathrm{o},i}^{\mathrm{r}}(\tau_i,\delta_i)] + \alpha^{\mathrm{Dep}}(T,p,\bar{x})$$









Finally, the departure function becomes

$$\alpha^{\text{Dep}}(T, p, \bar{x}) = \frac{\alpha^{\text{E}}(T, p, \bar{x})}{RT} - \sum_{i=1}^{N} x_i \ln\left(\frac{\rho}{\rho_i}\right) - \sum_{i=1}^{N} x_i [\alpha^{\text{r}}_{\text{o},i}(\tau, \delta) - \alpha^{\text{r}}_{\text{o},i}(\tau_i, \delta_i)]$$

Choice of a reference pressure:

- Like PSRK (Holderbaum and Gmehling (1991))
- Saturated liquid at $p_0 = 101325$ Pa
- Assumption of a constant packing fraction u at saturated liquid conditions







The following assumptions are used:

- The reference pressure is chosen as $p_0 = 101325$ Pa (see PSRK, VTPR)
- The densities of the liquid phase at p_0 are calculated assuming (see PSRK, VTPR)

u = 1.17 = const. Inverse packing fraction: v

$$u = \frac{v_{\rm s}^{\rm L}(p_0)}{b} = \frac{v_{{\rm s},i}^{\rm L}(p_0)}{b_i}$$

— The densities at the reference pressure then become

$$\rho_{\rm ref} = \frac{1}{ub} \quad \text{and} \quad \rho_{i,\rm ref} = \frac{1}{ub_i}$$







Calculated inverse packing fractions of saturated liquids at p_0











The inverse packing fraction is defined as

u - Inverse packing fraction

 $u = \frac{v^{L}}{h}$

- $v^{\rm L}$ Molar volume of the saturated liquid
- b covolume calculated with a cubic equation of state

For pure substances:
$$u_i = \frac{v_i^{L}}{b_i}$$
 with $b_i = 0.08664 \frac{RT_{c,i}}{p_{c,i}}$

The covolume of the mixture is calculated with a linear mixing rule $b = \sum x_i b_i$









With the definition of the inverse packing fraction follows

 $\rho_{\rm ref} = \frac{1}{ub}$ and $\rho_{i,\rm ref} = \frac{1}{ub_i}$ Densities of saturated liquid at p_0

Hence, the reduced densities at reference conditions become

$$\delta_{\rm ref} = \frac{1}{ub\rho_{\rm r}}$$
 and $\delta_{i,{\rm ref}} = \frac{1}{ub_i\rho_{{\rm c},i}}$

And therefore the departure function

$$\alpha^{\text{Dep}} = \frac{\alpha^{\text{E}}}{RT} - \sum_{i=1}^{N} x_i \ln\left(\frac{b_i}{b}\right) - \sum_{i=1}^{N} x_i [\alpha_{\text{o}i}^{\text{r}}(\delta_{\text{ref}}, \tau) - \alpha_{\text{o}i}^{\text{r}}(\delta_{i,\text{ref}}, \tau_i)]$$







The residual part should hold: $\lim_{\rho \to 0} \alpha^{r} = 0$. Therefore, a density dependent term is needed.

Translating the PSRK to the reduced residual Helmholtz energy, the following is obtained

$$\alpha_{\text{PSRK}}^{\text{r}} = -\ln(1-b\rho) + \frac{\ln(1+b\rho)}{\ln\left(1+\frac{1}{u}\right)} \begin{cases} g_{\text{GE}}^{\text{E}} - \sum_{i=1}^{N} x_i \ln\left(\frac{b_i}{b}\right) - \frac{1}{RT} \ln\left(1+\frac{1}{u}\right) \sum_{i=1}^{N} x_i \frac{a_i}{b_i} \end{cases}$$

Applying the density dependent term of the PSRK yields:

$$\alpha^{\text{Dep}} = \frac{\ln(1+b\rho)}{\ln\left(1+\frac{1}{u}\right)} \left\{ \frac{a_{\text{GE}}^{\text{E}}}{RT} - \sum_{i=1}^{N} x_i \ln\left(\frac{b_i}{b}\right) - \sum_{i=1}^{N} x_i [\alpha_{\text{o}i}^{\text{r}}(\delta_{\text{ref}},\tau) - \alpha_{\text{o}i}^{\text{r}}(\delta_{i,\text{ref}},\tau_i)] \right\}$$







Results for All Binary Mixtures With Linear Mixing Rules



10-2

For more results, see

Jäger, A.; Bell, I.H.; Breitkopf, C.: "A theoretically based departure function for multi-fluid mixture models", Fluid Phase Equilibria 469, 56-69, 2018.











— Linear mixing rules







- New model with fitted parameters









- New model with PSRK parameters









- New model with VTPR parameters







- Results of full PSRK









Results with Standard Mixing Rules

Results for ethane + propene and benzene + ethanol with **linear mixing rules**









Results of the New Model with UNIFAC (VTPR) Parameters

Results for ethane + propene and benzene + ethanol with VTPR parameters for UNIFAC









Results of the new model with COSMO-SAC (Mullins et al. 2006)

Results for ethane + propene und benzene + ethanol with COSMO-SAC









Results for mixtures with water: water + acetone

Results for acetone + water with **linear mixing rules**









Results for mixtures with water: water + acetone

Results for acetone + water with VTPR parameters for UNIFAC









Results for mixtures with water: water + acetone

Results for acetone + water with partly fitted VTPR parameters for UNIFAC



Fitted to data: *a*₇₉, *b*₇₉, *c*₇₉, *a*₉₇, *b*₉₇, *c*₉₇







